

sem.1 Relational models

int:sem:rel:
sec In order to give a precise semantics for intuitionistic propositional logic, we have to give a definition of what counts as a model relative to which we can evaluate **formulas**. On the basis of such a definition it is then also possible to define semantics notions such as validity and entailment. One such semantics is given by **relational models**.

Definition sem.1. A **relational model** for intuitionistic propositional logic is a triple $\mathfrak{M} = \langle W, R, V \rangle$, where

1. W is a non-empty set,
2. R is a partial order (i.e., a reflexive, antisymmetric, and transitive binary relation) on W , and
3. V is a function assigning to each **propositional variable** p a subset of W , such that
4. V is monotone with respect to R , i.e., if $w \in V(p)$ and Rww' , then $w' \in V(p)$.

int:sem:rel:
defn:true-at-w **Definition sem.2.** We define the notion of φ being true at w in \mathfrak{M} , $\mathfrak{M}, w \Vdash \varphi$, inductively as follows:

1. $\varphi \equiv p$: $\mathfrak{M}, w \Vdash \varphi$ iff $w \in V(p)$.
2. $\varphi \equiv \perp$: not $\mathfrak{M}, w \Vdash \varphi$.
3. $\varphi \equiv \neg\psi$: $\mathfrak{M}, w \Vdash \varphi$ iff for no w' such that Rww' , $\mathfrak{M}, w' \Vdash \psi$.
4. $\varphi \equiv \psi \wedge \chi$: $\mathfrak{M}, w \Vdash \varphi$ iff $\mathfrak{M}, w \Vdash \psi$ and $\mathfrak{M}, w \Vdash \chi$.
5. $\varphi \equiv \psi \vee \chi$: $\mathfrak{M}, w \Vdash \varphi$ iff $\mathfrak{M}, w \Vdash \psi$ or $\mathfrak{M}, w \Vdash \chi$ (or both).
6. $\varphi \equiv \psi \rightarrow \chi$: $\mathfrak{M}, w \Vdash \varphi$ iff for every w' such that Rww' , not $\mathfrak{M}, w' \Vdash \psi$ or $\mathfrak{M}, w' \Vdash \chi$ (or both).

We write $\mathfrak{M}, w \not\Vdash \varphi$ if not $\mathfrak{M}, w \Vdash \varphi$. If Γ is a set of **formulas**, $\mathfrak{M}, w \Vdash \Gamma$ means $\mathfrak{M}, w \Vdash \psi$ for all $\psi \in \Gamma$.

Problem sem.1. Show that according to **Definition sem.2**, $\mathfrak{M}, w \Vdash \neg\varphi$ iff $\mathfrak{M}, w \Vdash \varphi \rightarrow \perp$.

int:sem:rel:
prop:true-monotonic **Proposition sem.3.** Truth at worlds is monotonic with respect to R , i.e., if $\mathfrak{M}, w \Vdash \varphi$ and Rww' , then $\mathfrak{M}, w' \Vdash \varphi$.

Proof. Exercise. □

Problem sem.2. Prove **Proposition sem.3**.

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Bibliography