

## sem.1 Relational models

int:sem:rel:  
sec In order to give a precise semantics for intuitionistic propositional logic, we have to give a definition of what counts as a model relative to which we can evaluate **formulas**. On the basis of such a definition it is then also possible to define semantics notions such as validity and entailment. One such semantics is given by **relational models**.

**Definition sem.1.** A **relational model** for intuitionistic propositional logic is a triple  $\mathfrak{M} = \langle W, R, V \rangle$ , where

1.  $W$  is a non-empty set,
2.  $R$  is a reflexive and transitive binary relation on  $W$ , and
3.  $V$  is function assigning to each **propositional variable**  $p$  a subset of  $W$ , such that
4.  $V$  is monotone with respect to  $R$ , i.e., if  $w \in V(p)$  and  $Rww'$ , then  $w' \in V(p)$ .

int:sem:rel:  
defn:true-at-w **Definition sem.2.** We define the notion of  $\varphi$  being true at  $w$  in  $\mathfrak{M}$ ,  $\mathfrak{M}, w \Vdash \varphi$ , inductively as follows:

1.  $\varphi \equiv \perp$ : not  $\mathfrak{M}, w \Vdash \varphi$ .
2.  $\varphi \equiv \neg\psi$ :  $\mathfrak{M}, w \Vdash \varphi$  iff for every  $w'$  such that  $Rww'$ ,  $\mathfrak{M}, w \not\Vdash \psi$ .
3.  $\varphi \equiv \psi \wedge \chi$ :  $\mathfrak{M}, w \Vdash \varphi$  iff  $\mathfrak{M}, w \Vdash \psi$  and  $\mathfrak{M}, w \Vdash \chi$ .
4.  $\varphi \equiv \psi \vee \chi$ :  $\mathfrak{M}, w \Vdash \varphi$  iff  $\mathfrak{M}, w \Vdash \psi$  or  $\mathfrak{M}, w \Vdash \chi$  (or both).
5.  $\varphi \equiv \psi \rightarrow \chi$ :  $\mathfrak{M}, w \Vdash \varphi$  iff for every  $w'$  such that  $Rww'$ , either not  $\mathfrak{M}, w \Vdash \psi$  or  $\mathfrak{M}, w' \Vdash \chi$ .

We write  $\mathfrak{M}, w \not\Vdash \varphi$  if not  $\mathfrak{M}, w \Vdash \varphi$ . If  $\Gamma$  is a set of **formulas**,  $\mathfrak{M}, w \Vdash \Gamma$  means  $\mathfrak{M}, w \Vdash \psi$  for all  $\psi \in \Gamma$ .

**Problem sem.1.** Show that according to **Definition sem.2**,  $\mathfrak{M}, w \Vdash \neg\varphi$  iff  $\mathfrak{M}, w \Vdash \varphi \rightarrow \perp$ .

int:sem:rel:  
prop:true-monotonic **Proposition sem.3.** Truth at worlds is monotonic with respect to  $R$ , i.e., if  $\mathfrak{M}, w \Vdash \varphi$  and  $Rww'$ , then  $\mathfrak{M}, w' \Vdash \varphi$ .

*Proof.* Exercise. □

**Problem sem.2.** Prove **Proposition sem.3**.

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**Bibliography**