sem.1 Relational models

int:sem:rel: sec In order to give a precise semantics for intuitionistic propositional logic, we have to give a definition of what counts as a model relative to which we can evaluate formulas. On the basis of such a definition it is then also possible to define semantics notions such as validity and entailment. One such semantics is given by relational models.

Definition sem.1. A relational model for intuitionistic propositional logic is a triple $\mathfrak{M} = \langle W, R, V \rangle$, where

1. W is a non-empty set,
2. R is a reflexive and transitive binary relation on W, and
3. V is function assigning to each propositional variable p a subset of W, such that
4. V is monotone with respect to R, i.e., if $w \in V(p)$ and Rww', then $w' \in V(p)$.

int:sem:rel: defn:true-at-w **Definition sem.2.** We define the notion of φ *being true at* w *in* $\mathfrak{M}$, $\mathfrak{M}, w \Vdash \varphi$, inductively as follows:

1. $\varphi \equiv p$: $\mathfrak{M}, w \Vdash \varphi$ iff $w \in V(p)$.
2. $\varphi \equiv \bot$: not $\mathfrak{M}, w \Vdash \varphi$.
3. $\varphi \equiv \neg\psi$: $\mathfrak{M}, w \Vdash \varphi$ iff for no w' such that Rww', $\mathfrak{M}, w' \Vdash \psi$.
4. $\varphi \equiv \psi \land \chi$: $\mathfrak{M}, w \Vdash \varphi$ iff $\mathfrak{M}, w \Vdash \psi$ and $\mathfrak{M}, w \Vdash \chi$.
5. $\varphi \equiv \psi \lor \chi$: $\mathfrak{M}, w \Vdash \varphi$ iff $\mathfrak{M}, w \Vdash \psi$ or $\mathfrak{M}, w \Vdash \chi$ (or both).
6. $\varphi \equiv \psi \to \chi$: $\mathfrak{M}, w \Vdash \varphi$ iff for every w' such that Rww', not $\mathfrak{M}, w \Vdash \psi$ or $\mathfrak{M}, w \Vdash \chi$ (or both).

We write $\mathfrak{M}, w \nVdash \varphi$ if not $\mathfrak{M}, w \Vdash \varphi$. If Γ is a set of formulas, $\mathfrak{M}, w \Vdash \Gamma$ means $\mathfrak{M}, w \Vdash \psi$ for all $\psi \in \Gamma$.

Problem sem.1. Show that according to Definition sem.2, $\mathfrak{M}, w \Vdash \neg\varphi$ iff $\mathfrak{M}, w \Vdash \varphi \to \bot$.

int:sem:rel: prop:true-monotonic **Proposition sem.3.** *Truth at worlds is monotonic with respect to R, i.e., if $\mathfrak{M}, w \Vdash \varphi$ and Rww', then $\mathfrak{M}, w' \Vdash \varphi$.*

Proof. Exercise. □

Problem sem.2. Prove Proposition sem.3.

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Bibliography