

## sem.1 Introduction

int:sem:int:  
sec

No logic is satisfactorily described without a semantics, and intuitionistic logic is no exception. Whereas for classical logic, the semantics based on **valuations** is canonical, there are several competing semantics for intuitionistic logic. None of them are completely satisfactory in the sense that they give an intuitionistically acceptable account of the meanings of the connectives.

The semantics based on **relational models**, similar to the semantics for modal logics, is perhaps the most popular one. In this semantics, **propositional variables** are assigned to worlds, and these worlds are related by an accessibility relation. That relation is always a partial order, i.e., it is reflexive, symmetric, and transitive.

Intuitively, you might think of these worlds as states of knowledge or “evidentiary situations.” A state  $w'$  is accessible from  $w$  iff, for all we know,  $w'$  is a possible (future) state of knowledge, i.e., one that is compatible with what’s known at  $w$ . Once a proposition is known, it can’t become un-known, i.e., whenever  $\varphi$  is known at  $w$  and  $Rww'$ ,  $\varphi$  is known at  $w'$  as well. So “knowledge” is monotonic with respect to the accessibility relation.

If we define “ $\varphi$  is known” as in epistemic logic as “true in all epistemic alternatives,” then  $\varphi \wedge \psi$  is known at  $w$  if in all epistemic alternatives, both  $\varphi$  and  $\psi$  are known. But since knowledge is monotonic and  $R$  is reflexive, that means that  $\varphi \wedge \psi$  is known at  $w$  iff  $\varphi$  and  $\psi$  are known at  $w$ . For the same reason,  $\varphi \vee \psi$  is known at  $w$  iff at least one of them is known. So for  $\wedge$  and  $\vee$ , the truth conditions of the connectives coincide with those in classical logic.

The truth conditions for the conditional, however, differ from classical logic.  $\varphi \rightarrow \psi$  is known at  $w$  iff at no  $w'$  with  $Rww'$ ,  $\varphi$  is known without  $\psi$  also being known. This does not coincide with  $\varphi \rightarrow \psi$  being known at  $w$  iff  $\varphi$  is unknown or  $\psi$  is known. For if we now know neither  $\varphi$  nor  $\psi$ , there might be future epistemic states where we come to know  $\varphi$  without also coming to know  $\psi$ .

On this interpretation, we know  $\neg\varphi$  only if there is no possible future epistemic state in which we know  $\varphi$ . Here the idea is that if  $\varphi$  were knowable, then in some possible future epistemic state  $\varphi$  becomes known. Since we can’t know  $\perp$ , in that future epistemic state, we would know  $\varphi$  but not know  $\perp$ .

On this interpretation the principle of excluded middle fails. For there are some  $\varphi$  which we don’t yet know, but which we might come to know. For such an  $\varphi$ , both  $\varphi$  and  $\neg\varphi$  are unknown, so  $\varphi \vee \neg\varphi$  is not known. But we do know, e.g., that  $\neg(\varphi \wedge \neg\varphi)$ . For no future state in which we know both  $\varphi$  and  $\neg\varphi$  is possible, and we know this independently of whether or not we know  $\varphi$  or  $\neg\varphi$ .

**Relational models** are not the only available semantics for intuitionistic logic. The topological semantics is another: here propositions are interpreted as open sets in a topological space, and the connectives are interpreted as operations on these sets (e.g.,  $\wedge$  corresponds to intersection).

**Photo Credits**

**Bibliography**