No logic is satisfactorily described without a semantics, and intuitionistic logic
is no exception. Whereas for classical logic, the semantics based on valuations is
canonical, there are several competing semantics for intuitionistic logic. None of
them are completely satisfactory in the sense that they give an intuitionistically
acceptable account of the meanings of the connectives.

The semantics based on relational models, similar to the semantics for
modal logics, is perhaps the most popular one. In this semantics, propositional
variables are assigned to worlds, and these worlds are related by an
accessibility relation. That relation is always a partial order, i.e., it is reflexive,
antisymmetric, and transitive.

Intuitively, you might think of these worlds as states of knowledge or “evidentiary situations.” A state \( w' \) is accessible from \( w \) iff, for all we know, \( w' \) is
a possible (future) state of knowledge, i.e., one that is compatible with what’s
known at \( w \). Once a proposition is known, it can’t become un-known, i.e.,
whenever \( \varphi \) is known at \( w \) and \( Rww' \), \( \varphi \) is known at \( w' \) as well. So “knowledge” is monotonic with respect to the accessibility relation.

If we define “\( \varphi \) is known” as in epistemic logic as “true in all epistemic
alternatives,” then \( \varphi \land \psi \) is known at \( w \) if in all epistemic alternatives, both \( \varphi \)
and \( \psi \) are known. But since knowledge is monotonic and \( R \) is reflexive, that
means that \( \varphi \land \psi \) is known at \( w \) iff \( \varphi \) and \( \psi \) are known at \( w \). For the same
reason, \( \varphi \lor \psi \) is known at \( w \) iff at least one of them is known. So for \( \land \) and \( \lor \),
the truth conditions of the connectives coincide with those in classical logic.

The truth conditions for the conditional, however, differ from classical logic.
\( \varphi \rightarrow \psi \) is known at \( w \) iff at no \( w' \) with \( Rww' \), \( \varphi \) is known without \( \psi \) also being
known. This is not the same as the condition that \( \varphi \) is unknown or \( \psi \) is known
at \( w \). For if we know neither \( \varphi \) nor \( \psi \) at \( w \), there might be a future epistemic
state \( w' \) with \( Rww' \) such that at \( w' \), \( \varphi \) is known without also coming to know \( \psi \).

We know \( \neg \varphi \) only if there is no possible future epistemic state in which
we know \( \varphi \). Here the idea is that if \( \varphi \) were knowable, then in some possible
future epistemic state \( \varphi \) becomes known. Since we can’t know \( \bot \), in that future
epistemic state, we would know \( \varphi \) but not know \( \bot \).

On this interpretation the principle of excluded middle fails. For there are
some \( \varphi \) which we don’t yet know, but which we might come to know. For such
a formula \( \varphi \), both \( \varphi \) and \( \neg \varphi \) are unknown, so \( \varphi \lor \neg \varphi \) is not known. But we do
know, e.g., that \( \neg (\varphi \land \neg \varphi) \). For no future state in which we know both \( \varphi \) and
\( \neg \varphi \) is possible, and we know this independently of whether or not we know \( \varphi \)
or \( \neg \varphi \).

Relational models are not the only available semantics for intuitionistic
logic. The topological semantics is another: here propositions are interpreted
as open sets in a topological space, and the connectives are interpreted as
operations on these sets (e.g., \( \land \) corresponds to intersection).
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Bibliography