

pty.1 Proof Terms

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sec We give the definition of proof terms, and then establish its relation with natural deduction **derivations**.

Definition pty.1 (Proof terms). Proof terms are inductively generated by the following rules:

1. A single variable x is a proof term.
2. If P and Q are proof terms, then PQ is also a proof term.
3. If x is a variable, φ is a **formula**, and N is a proof term, then $\lambda x^\varphi. N$ is also a proof term.
4. If P and Q are proof terms, then $\langle P, Q \rangle$ is a proof term.
5. If M is a proof term, then $p_i(M)$ is also a proof term, where i is 1 or 2.
6. If M is a proof term, and φ is a formula, then $\text{in}_i^\varphi(M)$ is a proof term, where i is 1 or 2.
7. If M, N_1, N_2 is proof terms, and x_1, x_2 are variables, then $\text{case}(M, x_1.N_1, x_2.N_2)$ is a proof term.
8. If M is a proof term and φ is a formula, then $\text{contr}_\varphi(M)$ is proof term.

Each of the above rules corresponds to an inference rule in natural deduction. Thus we can inductively assign proof terms to the **formulas** in a **derivation**. To make this assignment unique, we must distinguish between the two versions of \wedge Elim and of \vee Intro. For instance, the proof terms assigned to the conclusion of \vee Intro must carry the information whether $\varphi \vee \psi$ is inferred from φ or from ψ . Suppose M is the term assigned to φ from which $\varphi \vee \psi$ is inferred. Then the proof term assigned to $\varphi \vee \psi$ is $\text{in}_1^\varphi(M)$. If we instead infer $\psi \vee \varphi$ then the proof term assigned is $\text{in}_2^\varphi(M)$.

The term $\lambda x^\varphi. N$ is assigned to the conclusion of \rightarrow Intro. The φ represents the assumption being discharged; only have we included it can we infer the formula of $\lambda x^\varphi. N$ based on the formula of N .

Definition pty.2 (Typing context). A *typing context* is a mapping from variables to formulas. We will call it simply the “context” if there is no confusion. We write a context Γ as a set of pairs $\langle x, \varphi \rangle$.

A pair $\Gamma \Rightarrow M$ where M is a proof term represents a **derivation** of a formula with context Γ .

Definition pty.3 (Typing pair). A *typing pair* is a pair $\langle \Gamma, M \rangle$, where Γ is a typing context and M is a proof term.

Since in general terms only make sense with specific contexts, we will speak simply of “terms” from now on instead of “typing pair”; and it will be apparent when we are talking about the literal term M .

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Bibliography