int.1 Syntax of Intuitionistic Logic

int:int:syn:sec

The syntax of intuitionistic logic is the same as that for propositional logic. In classical propositional logic it is possible to define connectives by others, e.g., one can define $\varphi \rightarrow \psi$ by $\neg\varphi \lor \psi$, or $\varphi \lor \psi$ by $\neg(\neg\varphi \land \neg\psi)$. Thus, presentations of classical logic often introduce some connectives as abbreviations for these definitions. This is not so in intuitionistic logic, with two exceptions: $\neg\varphi$ can be—and often is—defined as an abbreviation for $\varphi \rightarrow \bot$. Then, of course, $\bot$ must not itself be defined! Also, $\varphi \leftrightarrow \psi$ can be defined, as in classical logic, as $(\varphi \rightarrow \psi) \land (\psi \rightarrow \varphi)$.

Formulas of propositional intuitionistic logic are built up from *propositional variables* and the propositional constant $\bot$ using *logical connectives*. We have:

1. A denumerable set At_0 of propositional variables p_0, p_1, ...
2. The propositional constant for falsity $\bot$.
3. The logical connectives: $\land$ (conjunction), $\lor$ (disjunction), $\rightarrow$ (conditional)
4. Punctuation marks: (,), and the comma.

int:int:syn:defn:formulas

Definition int.1 (Formula)**.** The set $\mathrm{Frm}(\mathcal{L}_0)$ of *formulas* of propositional intuitionistic logic is defined inductively as follows:

1. $\bot$ is an atomic formula.
2. Every propositional variable p_i is an atomic formula.
3. If φ and ψ are formulas, then $(\varphi \land \psi)$ is a formula.
4. If φ and ψ are formulas, then $(\varphi \lor \psi)$ is a formula.
5. If φ and ψ are formulas, then $(\varphi \rightarrow \psi)$ is a formula.
6. Nothing else is a formula.

In addition to the primitive connectives introduced above, we also use the following *defined* symbols: $\neg$ (negation) and $\leftrightarrow$ (biconditional). Formulas constructed using the defined operators are to be understood as follows:

1. $\neg\varphi$ abbreviates $\varphi \rightarrow \bot$.
2. $\varphi \leftrightarrow \psi$ abbreviates $(\varphi \rightarrow \psi) \land (\psi \rightarrow \varphi)$.

Although $\neg$ is officially treated as an abbreviation, we will sometimes give explicit rules and clauses in definitions for $\neg$ as if it were primitive. This is mostly so we can state practice problems.

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Bibliography