Theorem tcp.2. Let $T$ be any $\omega$-consistent theory that includes $Q$. Then $T$ is not decidable.

Proof. If $T$ includes $Q$, then $T$ represents the computable functions and relations. We need only modify the previous proof. As above, if $x \in K$, then $T$ proves $\exists s \varphi_T(x, x, s)$. Conversely, suppose $T$ proves $\exists s \varphi_T(x, x, s)$. Then $x$ must be in $K$: otherwise, there is no halting computation of machine $x$ on input $x$; since $\varphi_T$ represents Kleene’s $T$ relation, $T$ proves $\neg\varphi_T(x, x, 0), \neg\varphi_T(x, x, 1), \ldots$, making $T$ $\omega$-inconsistent. $\square$

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Bibliography