Let \( \bar{\text{Q}} \) be the set of sentences whose \textit{negations} are provable in \( \text{Q} \), i.e., \( \bar{\text{Q}} = \{ \varphi : \text{Q} \vdash \neg \varphi \} \). Remember that disjoint sets \( \text{A} \) and \( \text{B} \) are said to be computably inseparable if there is no computable set \( \text{C} \) such that \( \text{A} \subseteq \text{C} \) and \( \text{B} \subseteq \overline{\text{C}} \).

\[ \text{Lemma tcp.1. } \text{Q} \text{ and } \bar{\text{Q}} \text{ are computably inseparable.} \]

\[ \text{Proof.} \text{ Suppose } \text{C} \text{ is a computable set such that } \text{Q} \subseteq \text{C} \text{ and } \bar{\text{Q}} \subseteq \overline{\text{C}} \text{. Let } R(x, y) \text{ be the relation} \]
\[ x \text{ codes a formula } \theta(u) \text{ and } \theta(\overline{u}) \text{ is in } \text{C}. \]

We will show that \( R(x, y) \) is a universal computable relation, yielding a contradiction.

Suppose \( S(y) \) is computable, represented by \( \theta_S(u) \) in \( \text{Q} \). Then
\[
S(\overline{u}) \rightarrow \text{Q} \vdash \theta_S(\overline{u}) \\
\rightarrow \theta_S(\overline{u}) \in \text{C}
\]

and
\[
\neg S(\overline{u}) \rightarrow \text{Q} \vdash \neg \theta_S(\overline{u}) \\
\rightarrow \theta_S(\overline{u}) \in \bar{\text{Q}} \\
\rightarrow \theta_S(\overline{u}) \notin \text{C}
\]

So \( S(y) \) is equivalent to \( R(\#(\theta_S(\overline{u})), y) \).

\[ \square \]

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\[ \text{Bibliography} \]