

tcp.1 **Axiomatizable Theories**

inc:tcp:ax:
sec A theory \mathbf{T} is said to be *axiomatizable* if it has a computable set of axioms A . (Saying that A is a set of axioms for \mathbf{T} means $T = \{\varphi : A \vdash \varphi\}$.) Any “reasonable” axiomatization of the natural numbers will have this property. In particular, any theory with a finite set of axioms is *axiomatizable*.

Lemma tcp.1. *Suppose \mathbf{T} is axiomatizable. Then \mathbf{T} is computably enumerable.*

Proof. Suppose A is a computable set of axioms for \mathbf{T} . To determine if $\varphi \in T$, just search for a proof of φ from the axioms.

Put slightly differently, φ is in \mathbf{T} if and only if there is a finite list of axioms ψ_1, \dots, ψ_k in A and a proof of $(\psi_1 \wedge \dots \wedge \psi_k) \rightarrow \varphi$ in first-order logic. But we already know that any set with a definition of the form “there exists ... such that ...” is *c.e.*, provided the second “...” is computable. \square

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Bibliography