## tcp.1 Axiomatizable Complete Theories are Decidable

 $_{\rm sec}^{\rm inc:tcp:cdc:}$ 

A theory is said to be *complete* if for every sentence  $\varphi$ , either  $\varphi$  or  $\neg \varphi$  is provable.

**Lemma tcp.1.** Suppose a theory T is complete and axiomatizable. Then T is decidable.

*Proof.* Suppose  $\mathbf{T}$  is complete and A is a computable set of axioms. If  $\mathbf{T}$  is inconsistent, it is clearly computable. (Algorithm: "just say yes.") So we can assume that  $\mathbf{T}$  is also consistent.

To decide whether or not a sentence  $\varphi$  is in **T**, simultaneously search for a proof of  $\varphi$  from A and a proof of  $\neg \varphi$ . Since **T** is complete, you are bound to find one or another; and since **T** is consistent, if you find a proof of  $\neg \varphi$ , there is no proof of  $\varphi$ .

Put in different terms, we already know that **T** is c.e.; so by a theorem we proved before, it suffices to show that the complement of **T** is c.e. also. But a formula  $\varphi$  is in  $\bar{\mathbf{T}}$  if and only if  $\neg \varphi$  is in  $\bar{\mathbf{T}}$ ; so  $\bar{\mathbf{T}} \leq_m \mathbf{T}$ .

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Bibliography