

tcp.1 **Axiomatizable** Complete Theories are Decidable

inc:tcp:edc:
sec A theory is said to be *complete* if for every sentence φ , either φ or $\neg\varphi$ is provable.

Lemma tcp.1. *Suppose a theory \mathbf{T} is complete and **axiomatizable**. Then \mathbf{T} is decidable.*

Proof. Suppose \mathbf{T} is complete and A is a computable set of axioms. If \mathbf{T} is inconsistent, it is clearly computable. (Algorithm: “just say yes.”) So we can assume that \mathbf{T} is also consistent.

To decide whether or not a sentence φ is in \mathbf{T} , simultaneously search for a **derivation** of φ from \mathbf{T} and a **derivation** of $\neg\varphi$. Since \mathbf{T} is complete, you are bound to find one or the other; and since \mathbf{T} is consistent, if you find a **derivation** of $\neg\varphi$, there is no **derivation** of φ .

Put in different terms, we already know that \mathbf{T} is **c.e.**; so by a theorem we proved before, it suffices to show that the complement of \mathbf{T} is **c.e.** also. But a **formula** φ is in $\bar{\mathbf{T}}$ if and only if $\neg\varphi$ is in \mathbf{T} ; so $\bar{\mathbf{T}} \leq_m \mathbf{T}$. \square

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Bibliography