

art.1 Substitution

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[inc:art:sub:](#) **Proposition art.1.** *There is a primitive recursive function $\text{Subst}(x, y, z)$ with the property that*

$$\text{Subst}(\ulcorner \varphi \urcorner, \ulcorner t \urcorner, \ulcorner u \urcorner) = \ulcorner \varphi[t/u] \urcorner$$

Proof. We can then define a function hSubst by primitive recursion as follows:

$$\begin{aligned} \text{hSubst}(x, y, z, 0) &= \emptyset \\ \text{hSubst}(x, y, z, i + 1) &= \begin{cases} \text{hSubst}(x, y, z, i) \smallfrown y & \text{if } \text{FreeOcc}(x, z, i + 1) \\ \text{append}(\text{hSubst}(x, y, z, i), (x)_{i+1}) & \text{otherwise.} \end{cases} \end{aligned}$$

$\text{Subst}(x, y, z)$ can now be defined as $\text{hSubst}(x, y, z, \text{len}(x))$. □

[inc:art:sub:](#) **Proposition art.2.** *The relation $\text{FreeFor}(x, y, z)$, which holds iff the term with Gödel number y is **free** for the variable with Gödel number z in the formula with Gödel number x , is primitive recursive.*

Proof. Exercise. □

Problem art.1. Prove [Proposition art.2](#)

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Bibliography