

art.1 Substitution

inc:art:sub:
sec Recall that substitution is the operation of replacing all free occurrences of a variable u in a formula φ by a term t , written $\varphi[t/u]$. This operation, when carried out on Gödel numbers of variables, formulas, and terms, is primitive recursive.

inc:art:sub:
prop:subst-primrec **Proposition art.1.** *There is a primitive recursive function $\text{Subst}(x, y, z)$ with the property that*

$$\text{Subst}(\# \varphi \#, \# t \#, \# u \#) = \# \varphi[t/u] \#$$

Proof. We can then define a function hSubst by primitive recursion as follows:

$$\begin{aligned} \text{hSubst}(x, y, z, 0) &= A \\ \text{hSubst}(x, y, z, i + 1) &= \begin{cases} \text{hSubst}(x, y, z, i) \frown y & \text{if } \text{FreeOcc}(x, z, i) \\ \text{append}(\text{hSubst}(x, y, z, i), (x)_i) & \text{otherwise.} \end{cases} \end{aligned}$$

$\text{Subst}(x, y, z)$ can now be defined as $\text{hSubst}(x, y, z, \text{len}(x))$. □

inc:art:sub:
prop:free-for **Proposition art.2.** *The relation $\text{FreeFor}(x, y, z)$, which holds iff the term with Gödel number y is free for the variable with Gödel number z in the formula with Gödel number x , is primitive recursive.*

Proof. Exercise. □

Problem art.1. Prove [Proposition art.2](#)

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Bibliography