art.1 Substitution

inc:art:sub:sec Recall that substitution is the operation of replacing all free occurrences of a variable u in a formula φ by a term t, written $\varphi[t/u]$. This operation, when carried out on Gödel numbers of variables, formulas, and terms, is primitive recursive.

inc:art:sub:prop:subst-primrec **Proposition art.1.** *There is a primitive recursive function* $\mathrm{Subst}(x, y, z)$ *with the property that*

$$\mathrm{Subst}({}^{\#}\varphi^{\#}, {}^{\#}t^{\#}, {}^{\#}u^{\#}) = {}^{\#}\varphi[t/u]^{\#}.$$

Proof. We can then define a function hSubst by primitive recursion as follows:

$$\begin{aligned}
\mathrm{hSubst}(x, y, z, 0) &= \Lambda \\
\mathrm{hSubst}(x, y, z, i+1) &= \\
&\begin{cases} \mathrm{hSubst}(x, y, z, i) \frown y & \text{if } \mathrm{FreeOcc}(x, z, i) \\ \mathrm{append}(\mathrm{hSubst}(x, y, z, i), (x)_i) & \text{otherwise.} \end{cases}
\end{aligned}$$

$\mathrm{Subst}(x, y, z)$ can now be defined as $\mathrm{hSubst}(x, y, z, \mathrm{len}(x))$. □

inc:art:sub:prop:free-for **Proposition art.2.** *The relation* $\mathrm{FreeFor}(x, y, z)$*, which holds iff the term with Gödel number* y *is free for the variable with Gödel number* z *in the formula with Gödel number* x*, is primitive recursive.*

Proof. Exercise. □

Problem art.1. Prove Proposition art.2

Photo Credits

Bibliography