

art.1 Coding Formulas

inc:art:frm:
sec

Proposition art.1. *The relation $\text{Atom}(x)$ which holds iff x is the Gödel number of an atomic formula, is primitive recursive.*

Proof. The number x is the Gödel number of an atomic formula iff one of the following holds:

1. There are $n, j < x$, and $z < x$ such that for each $i < n$, $\text{Term}((z)_i)$ and $x =$

$$\#P_j^n(\# \frown \text{flatten}(z) \frown \#)\#.$$

2. There are $z_1, z_2 < x$ such that $\text{Term}(z_1)$, $\text{Term}(z_2)$, and $x =$

$$\#=(\# \frown z_1 \frown \#, \# \frown z_2 \frown \#)\#.$$

3. $x = \# \perp \#$.

4. $x = \# \top \#$.

□

inc:art:frm:
prop:frm-primrec

Proposition art.2. *The relation $\text{Frm}(x)$ which holds iff x is the Gödel number of a formula is primitive recursive.*

Proof. A sequence of symbols s is a formula iff there is formation sequence $s_0, \dots, s_{k-1} = s$ of formula which records how s was formed from atomic formulas according to the formation rules. The code for each s_i (and indeed of the code of the sequence $\langle s_0, \dots, s_{k-1} \rangle$) is less than the code x of s . □

Problem art.1. Give a detailed proof of Proposition art.2 along the lines of the first proof of ??

Problem art.2. Give a detailed proof of Proposition art.2 along the lines of the alternate proof of ??

inc:art:frm:
prop:freeocc-primrec

Proposition art.3. *The relation $\text{FreeOcc}(x, z, i)$, which holds iff the i -th symbol of the formula with Gödel number x is a free occurrence of the variable with Gödel number z , is primitive recursive.*

Proof. Exercise. □

Problem art.3. Prove Proposition art.3. You may make use of the fact that any substring of a formula which is a formula is a sub-formula of it.

Proposition art.4. *The property $\text{Sent}(x)$ which holds iff x is the Gödel number of a sentence is primitive recursive.*

Proof. A **sentence** is a **formula** without free occurrences of **variables**. So $\text{Sent}(x)$ holds iff

$$(\forall i < \text{len}(x)) (\forall z < x) ((\exists j < z) z = \#v_j\# \rightarrow \neg \text{FreeOcc}(x, z, i)).$$

□

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Bibliography