

Part I
History

Chapter 1

Biographies

1.1 Georg Cantor

his:bio:can:
sec

An early biography of Georg Cantor (GAY-org KAHN-tor) claimed that he was born and found on a ship that was sailing for Saint Petersburg, Russia, and that his parents were unknown. This, however, is not true; although he was born in Saint Petersburg in 1845.

Cantor received his doctorate in mathematics at the University of Berlin in 1867. He is known for his work in set theory, and is credited with founding set theory as a distinctive research discipline. He was the first to prove that there are infinite sets of different sizes. His theories, and especially his theory of infinities, caused much debate among mathematicians at the time, and his work was controversial.

Cantor's religious beliefs and his mathematical work were inextricably tied; he even claimed that the theory of transfinite numbers had been communicated to him directly by God. In later life, Cantor suffered from mental illness. Beginning in 1884, and more frequently towards his later years, Cantor was hospitalized. The heavy criticism of his work, including a falling out with the mathematician Leopold Kronecker, led to depression and a lack of interest in mathematics. During depressive episodes, Cantor would turn to philosophy and literature, and even published a theory that Francis Bacon was the author of Shakespeare's plays.

Cantor died on January 6, 1918, in a sanatorium in Halle.



Figure 1.1: Georg Cantor

Further Reading For full biographies of Cantor, see [Dauben \(1990\)](#) and [Grattan-Guinness \(1971\)](#). Cantor’s radical views are also described in the BBC Radio 4 program *A Brief History of Mathematics* ([du Sautoy, 2014](#)). If you’d like to hear about Cantor’s theories in rap form, see [Rose \(2012\)](#).

1.2 Alonzo Church

Alonzo Church was born in Washington, DC on June 14, 1903. In early childhood, an air gun incident left Church blind in one eye. He finished preparatory school in Connecticut in 1920 and began his university education at Princeton that same year. He completed his doctoral studies in 1927. After a couple years abroad, Church returned to Princeton. Church was known exceedingly polite and careful. His blackboard writing was immaculate, and he would preserve important papers by carefully covering them in Duco cement. Outside of his academic pursuits, he enjoyed reading science fiction magazines and was not afraid to write to the editors if he spotted any inaccuracies in the writing.



[his:bio:church](#)

Figure 1.2: Alonzo Church

Church’s academic achievements were great. Together with his students Stephen Kleene and Barkley Rosser, he developed a theory of effective calculability, the lambda calculus, independently of Alan Turing’s development of the Turing machine. The two definitions of computability are equivalent, and give rise to what is now known as the *Church-Turing Thesis*, that a function of the natural numbers is effectively computable if and only if it is computable via Turing machine (or lambda calculus). He also proved what is now known as *Church’s Theorem*: The decision problem for the validity of first-order formulas is unsolvable.

Church continued his work into old age. In 1967 he left Princeton for UCLA, where he was professor until his retirement in 1990. Church passed away on August 1, 1995 at the age of 92.

Further Reading For a brief biography of Church, see [Enderton \(forthcoming\)](#). Church’s original writings on the lambda calculus and the Entscheidungsproblem (Church’s Thesis) are [Church \(1936a,b\)](#). [Aspray \(1984\)](#) records an interview with Church about the Princeton mathematics community in the 1930s. Church wrote a series of book reviews of the *Journal of Symbolic Logic* from 1936 until 1979. They are all archived on John MacFarlane’s website ([MacFarlane, 2015](#)).

1.3 Gerhard Gentzen

his:bio:gen:
sec

Gerhard Gentzen is known primarily as the creator of structural proof theory, and specifically the creation of the natural deduction and sequent calculus proof systems. He was born on November 24, 1909 in Greifswald, Germany. Gerhard was homeschooled for three years before attending preparatory school, where he was behind most of his classmates in terms of education. Despite this, he was a brilliant student and showed a strong aptitude for mathematics. His interests were varied, and he, for instance, also wrote poems for his mother and plays for the school theatre.

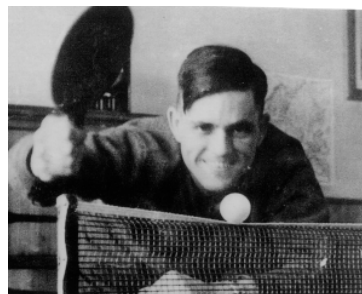


Figure 1.3: Gerhard Gentzen

Gentzen began his university studies at the University of Greifswald, but moved around to Göttingen, Munich, and Berlin. He received his doctorate in 1933 from the University of Göttingen under Hermann Weyl. (Paul Bernays supervised most of his work, but was dismissed from the university by the Nazis.) In 1934, Gentzen began work as an assistant to David Hilbert. That same year he developed the sequent calculus and natural deduction proof systems, in his papers *Untersuchungen über das logische Schließen I–II* [*Investigations Into Logical Deduction I–II*]. He proved the consistency of the Peano axioms in 1936.

Gentzen's relationship with the Nazis is complicated. At the same time his mentor Bernays was forced to leave Germany, Gentzen joined the university branch of the SA, the Nazi paramilitary organization. Like many Germans, he was a member of the Nazi party. During the war, he served as a telecommunications officer for the air intelligence unit. However, in 1942 he was released from duty due to a nervous breakdown. It is unclear whether or not Gentzen's loyalties lay with the Nazi party, or whether he joined the party in order to ensure academic success.

In 1943, Gentzen was offered an academic position at the Mathematical Institute of the German University of Prague, which he accepted. However, in 1945 the citizens of Prague revolted against German occupation. Soviet forces arrived in the city and arrested all the professors at the university. Because of his membership in Nazi organizations, Gentzen was taken to a forced labour camp. He died of malnutrition while in his cell on August 4, 1945 at the age of 35.

Further Reading For a full biography of Gentzen, see [Menzler-Trott \(2007\)](#). An interesting read about mathematicians under Nazi rule, which gives a brief note about Gentzen's life, is given by [Segal \(2014\)](#). Gentzen's papers on logical deduction are available in the original German ([Gentzen, 1935a,b](#)). English

translations of Gentzen’s papers have been collected in a single volume by Szabo (1969), which also includes a biographical sketch.

1.4 Kurt Gödel

Kurt Gödel (GER-dle) was born on April 28, 1906 in Brünn in the Austro-Hungarian empire (now Brno in the Czech Republic). Due to his inquisitive and bright nature, young Kurt was often called “Der kleine Herr Warum” (Little Mr. Why) by his family. He excelled in academics from primary school onward, where he got less than the highest grade only in mathematics. Gödel was often absent from school due to poor health and was exempt from physical education. He was diagnosed with rheumatic fever during his childhood. Throughout his life, he believed this permanently affected his heart despite medical assessment saying otherwise.



[his:bio:god:sec](#)

Figure 1.4: Kurt Gödel

Gödel began studying at the University of Vienna in 1924 and completed his doctoral studies in 1929. He first intended to study physics, but his interests soon moved to mathematics and especially logic, in part due to the influence of the philosopher Rudolf Carnap. His dissertation, written under the supervision of Hans Hahn, proved the completeness theorem of first-order predicate logic with identity (Gödel, 1929). Only a year later, he obtained his most famous results—the first and second incompleteness theorems (published in Gödel 1931). During his time in Vienna, Gödel was heavily involved with the Vienna Circle, a group of scientifically-minded philosophers that included Carnap, whose work was especially influenced by Gödel’s results.

In 1938, Gödel married Adele Nimbusky. His parents were not pleased: not only was she six years older than him and already divorced, but she worked as a dancer in a nightclub. Social pressures did not affect Gödel, however, and they remained happily married until his death.

After Nazi Germany annexed Austria in 1938, Gödel and Adele emigrated to the United States, where he took up a position at the Institute for Advanced Study in Princeton, New Jersey. Despite his introversion and eccentric nature, Gödel’s time at Princeton was collaborative and fruitful. He published essays in set theory, philosophy and physics. Notably, he struck up a particularly strong friendship with his colleague at the IAS, Albert Einstein.

In his later years, Gödel’s mental health deteriorated. His wife’s hospitalization in 1977 meant she was no longer able to cook his meals for him. Having suffered from mental health issues throughout his life, he succumbed to paranoia. Deathly afraid of being poisoned, Gödel refused to eat. He died of starvation on January 14, 1978, in Princeton.

Further Reading For a complete biography of Gödel’s life is available, see [John Dawson \(1997\)](#). For further biographical pieces, as well as essays about Gödel’s contributions to logic and philosophy, see [Wang \(1990\)](#), [Baaz et al. \(2011\)](#), [Takeuti et al. \(2003\)](#), and [Sigmund et al. \(2007\)](#).

Gödel’s PhD thesis is available in the original German ([Gödel, 1929](#)). The original text of the incompleteness theorems is ([Gödel, 1931](#)). All of Gödel’s published and unpublished writings, as well as a selection of correspondence, are available in English in his *Collected Papers* [Feferman et al. \(1986, 1990\)](#).

For a detailed treatment of Gödel’s incompleteness theorems, see [Smith \(2013\)](#). For an informal, philosophical discussion of Gödel’s theorems, see Mark Linsenmayer’s podcast ([Linsenmayer, 2014](#)).

1.5 Emmy Noether

[his:bio:noe:](#)
[sec](#)

Emmy Noether (NER-ter) was born in Erlangen, Germany, on March 23, 1882, to an upper-middle class scholarly family. Hailed as the “mother of modern algebra,” Noether made groundbreaking contributions to both mathematics and physics, despite significant barriers to women’s education. In Germany at the time, young girls were meant to be educated in arts and were not allowed to attend college preparatory schools. However, after auditing classes at the Universities of Göttingen and Erlangen (where her father was professor of mathematics), Noether was eventually able to enrol as a student at Erlangen in 1904, when their policy was updated to allow female students. She received her doctorate in mathematics in 1907.



Figure 1.5: Emmy Noether

Despite her qualifications, Noether experienced much resistance during her career. From 1908–1915, she taught at Erlangen without pay. During this time, she caught the attention of David Hilbert, one of the world’s foremost mathematicians of the time, who invited her to Göttingen. However, women were prohibited from obtaining professorships, and she was only able to lecture

under Hilbert’s name, again without pay. During this time she proved what is now known as Noether’s theorem, which is still used in theoretical physics today. Noether was finally granted the right to teach in 1919. Hilbert’s response to continued resistance of his university colleagues reportedly was: “Gentlemen, the faculty senate is not a bathhouse.”

In the later 1920s, she concentrated on work in abstract algebra, and her contributions revolutionized the field. In her proofs she often made use of the so-called ascending chain condition, which states that there is no infinite strictly increasing chain of certain sets. For instance, certain algebraic structures now known as Noetherian rings have the property that there are no infinite sequences of ideals $I_1 \subsetneq I_2 \subsetneq \dots$. The condition can be generalized to any partial order (in algebra, it concerns the special case of ideals ordered by the subset relation), and we can also consider the dual descending chain condition, where every strictly *decreasing* sequence in a partial order eventually ends. If a partial order satisfies the descending chain condition, it is possible to use induction along this order in a similar way in which we can use induction along the $<$ order on \mathbb{N} . Such orders are called *well-founded* or *Noetherian*, and the corresponding proof principle *Noetherian induction*.

Noether was Jewish, and when the Nazis came to power in 1933, she was dismissed from her position. Luckily, Noether was able to emigrate to the United States for a temporary position at Bryn Mawr, Pennsylvania. During her time there she also lectured at Princeton, although she found the university to be unwelcoming to women (Dick, 1981, 81). In 1935, Noether underwent an operation to remove a uterine tumour. She died from an infection as a result of the surgery, and was buried at Bryn Mawr.

Further Reading For a biography of Noether, see Dick (1981). The Perimeter Institute for Theoretical Physics has their lectures on Noether’s life and influence available online (Institute, 2015). If you’re tired of reading, *Stuff You Missed in History Class* has a podcast on Noether’s life and influence (Frey and Wilson, 2015). The collected works of Noether are available in the original German (Jacobson, 1983).

1.6 Rózsa Péter

Rózsa Péter was born Rózsa Politzer, in Budapest, Hungary, on February 17, 1905. She is best known for her work on recursive functions, which was essential for the creation of the field of recursion theory.

Péter was raised during harsh political times—WWI raged when she was a teenager—but was able to attend the affluent Maria Terezia Girls’ School in Budapest, from where she graduated in 1922. She then studied at Pázmány Péter University (later renamed Loránd Eötvös University) in Budapest. She began studying chemistry at the insistence of her father, but later switched to mathematics, and graduated in 1927. Although she had the credentials to teach high school mathematics, the economic situation at the time was dire as the

[his:bio:pet:
sec](#)

Great Depression affected the world economy. During this time, Péter took odd jobs as a tutor and private teacher of mathematics. She eventually returned to university to take up graduate studies in mathematics. She had originally planned to work in number theory, but after finding out that her results had already been proven, she almost gave up on mathematics altogether. She was encouraged to work on Gödel's incompleteness theorems, and unknowingly proved several of his results in different ways. This restored her confidence, and Péter went on to write her first papers on recursion theory, inspired by David Hilbert's foundational program. She received her PhD in 1935, and in 1937 she became an editor for the *Journal of Symbolic Logic*.

Péter's early papers are widely credited as founding contributions to the field of recursive function theory. In Péter (1935a), she investigated the relationship between different kinds of recursion. In Péter (1935b), she showed that a certain recursively defined function is not primitive recursive. This simplified an earlier result due to Wilhelm Ackermann. Péter's simplified function is what's now often called the Ackermann function—and sometimes, more properly, the Ackermann-Péter function. She wrote the first book on recursive function theory (Péter, 1951).

Despite the importance and influence of her work, Péter did not obtain a full-time teaching position until 1945.

During the Nazi occupation of Hungary during World War II, Péter was not allowed to teach due to anti-Semitic laws. In 1944 the government created a Jewish ghetto in Budapest; the ghetto was cut off from the rest of the city and attended by armed guards. Péter was forced to live in the ghetto until 1945 when it was liberated. She then went on to teach at the Budapest Teachers Training College, and from 1955 onward at Eötvös Loránd University. She was the first female Hungarian mathematician to become an Academic Doctor of Mathematics, and the first woman to be elected to the Hungarian Academy of Sciences.

Péter was known as a passionate teacher of mathematics, who preferred to explore the nature and beauty of mathematical problems with her students rather than to merely lecture. As a result, she was affectionately called “Aunt Rosa” by her students. Péter died in 1977 at the age of 71.



Figure 1.6: Rózsa Péter

Further Reading For more biographical reading, see (O'Connor and Robertson, 2014) and (Andrásfai, 1986). Tamassy (1994) conducted a brief interview with Péter. For a fun read about mathematics, see Péter's book *Playing With*

Infinity (Péter, 2010).

1.7 Julia Robinson

Julia Bowman Robinson was an American mathematician. She is known mainly for her work on decision problems, and most famously for her contributions to the solution of Hilbert's tenth problem. Robinson was born in St. Louis, Missouri on December 8, 1919. At a young age Robinson recalls being intrigued by numbers (Reid, 1986, 4). At age nine she contracted scarlet fever and suffered from several recurrent bouts of rheumatic fever. This forced her to spend much of her time in bed, putting her behind in her education. Although she was able to catch up with the help of private tutors, the physical effects of her illness had a lasting impact on her life.

Despite her childhood struggles, Robinson graduated high school with several awards in mathematics and the sciences. She started her university career at San Diego State College, and transferred to the University of California, Berkeley as a senior. There she was highly influenced by mathematician Raphael Robinson. They quickly became good friends, and married in 1941. As a spouse of a faculty member, Robinson was barred from teaching in the mathematics department at Berkeley. Although she continued to audit mathematics classes, she hoped to leave university and start a family. Not long after her wedding, however, Robinson contracted pneumonia. She was told that there was substantial scar tissue build up on her heart due to the rheumatic fever she suffered as a child. Due to the severity of the scar tissue, the doctor predicted that she would not live past forty and she was advised not to have children (Reid, 1986, 13).

Robinson was depressed for a long time, but eventually decided to continue studying mathematics. She returned to Berkeley and completed her PhD in 1948 under the supervision of Alfred Tarski. The first-order theory of the real numbers had been shown to be decidable by Tarski, and from Gödel's work it followed that the first-order theory of the natural numbers is undecidable. It was a major open problem whether the first-order theory of the rationals is decidable or not. In her thesis (1949), Robinson proved that it was not.

Interested in decision problems, Robinson next attempted to find a solution Hilbert's tenth problem. This problem was one of a famous list of 23 mathematical problems posed by David Hilbert in 1900. The tenth problem



Figure 1.7: Julia Robinson

[his:bio:rob:sec](#)

asks whether there is an algorithm that will answer, in a finite amount of time, whether or not a polynomial equation with integer coefficients, such as $3x^2 - 2y + 3 = 0$, has a solution in the integers. Such questions are known as *Diophantine problems*. After some initial successes, Robinson joined forces with Martin Davis and Hilary Putnam, who were also working on the problem. They succeeded in showing that exponential Diophantine problems (where the unknowns may also appear as exponents) are undecidable, and showed that a certain conjecture (later called “J.R.”) implies that Hilbert’s tenth problem is undecidable (Davis et al., 1961). Robinson continued to work on the problem for the next decade. In 1970, the young Russian mathematician Yuri Matijasevich finally proved the J.R. hypothesis. The combined result is now called the Matijasevich-Robinson-Davis-Putnam theorem, or MDRP theorem for short. Matijasevich and Robinson became friends and collaborated on several papers. In a letter to Matijasevich, Robinson once wrote that “actually I am very pleased that working together (thousands of miles apart) we are obviously making more progress than either one of us could alone” (Matijasevich, 1992, 45).

Robinson was the first female president of the American Mathematical Society, and the first woman to be elected to the National Academy of Science. She died on July 30, 1985 at the age of 65 after being diagnosed with leukemia.

Further Reading Robinson’s mathematical papers are available in her *Collected Works* (Robinson, 1996), which also includes a reprint of her National Academy of Sciences biographical memoir (Feferman, 1994). Robinson’s older sister Constance Reid published an “Autobiography of Julia,” based on interviews (Reid, 1986), as well as a full memoir (Reid, 1996). A short documentary about Robinson and Hilbert’s tenth problem was directed by George Csicsery (Csicsery, 2016). For a brief memoir about Yuri Matijasevich’s collaborations with Robinson, and her influence on his work, see (Matijasevich, 1992).

1.8 Bertrand Russell

his:bio:rus:
sec

Bertrand Russell is hailed as one of the founders of modern analytic philosophy. Born May 18, 1872, Russell was not only known for his work in philosophy and logic, but wrote many popular books in various subject areas. He was also an ardent political activist throughout his life.

Russell was born in Trellech, Monmouthshire, Wales. His parents were members of the British nobility. They were free-thinkers, and even made friends with the radicals in Boston at the time. Unfortunately, Russell’s parents died when he was young, and Russell was sent to live with his grandparents. There, he was given a religious upbringing (something his parents had wanted to avoid at all costs). His grandmother was very strict in all matters of morality. During adolescence he was mostly homeschooled by private tutors.

Russell’s influence in analytic philosophy, and especially logic, is tremendous. He studied mathematics and philosophy at Trinity College, Cambridge,

where he was influenced by the mathematician and philosopher Alfred North Whitehead. In 1910, Russell and Whitehead published the first volume of *Principia Mathematica*, where they championed the view that mathematics is reducible to logic. He went on to publish hundreds of books, essays and political pamphlets. In 1950, he won the Nobel Prize for literature.

Russell's was deeply entrenched in politics and social activism. During World War I he was arrested and sent to prison for six months due to pacifist activities and protest. While in prison, he was able to write and read, and claims to have found the experience "quite agreeable." He remained a pacifist throughout his life, and was again incarcerated for attending a nuclear disarmament rally in 1961. He also survived a plane crash in 1948, where the only survivors were those sitting in the smoking section. As such, Russell claimed that he owed his life to smoking. Russell was married four times, but had a reputation for carrying on extra-marital affairs. He died on February 2, 1970 at the age of 97 in Penrhyndeudraeth, Wales.

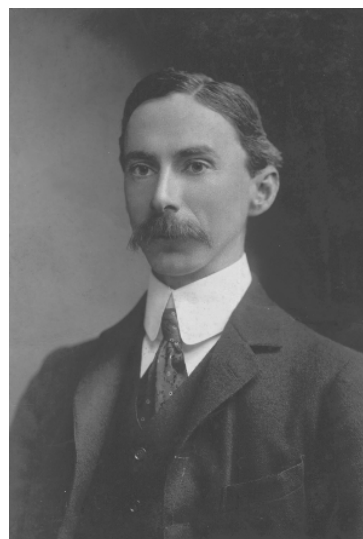


Figure 1.8: Bertrand Russell

Further Reading Russell wrote an autobiography in three parts, spanning his life from 1872–1967 (Russell, 1967, 1968, 1969). The Bertrand Russell Research Centre at McMaster University is home of the Bertrand Russell archives. See their website at Duncan (2015), for information on the volumes of his collected works (including searchable indexes), and archival projects. Russell's paper *On Denoting* (Russell, 1905) is a classic of 20th century analytic philosophy.

The Stanford Encyclopedia of Philosophy entry on Russell (Irvine, 2015) has sound clips of Russell speaking on Desire and Political theory. Many video interviews with Russell are available online. To see him talk about smoking and being involved in a plane crash, e.g., see Russell (n.d.). Some of Russell's works, including his *Introduction to Mathematical Philosophy* are available as free audiobooks on LibriVox (n.d.).

1.9 Alfred Tarski

Alfred Tarski was born on January 14, 1901 in Warsaw, Poland (then part of the Russian Empire). Often described as "Napoleonic," Tarski was boisterous, talkative, and intense. His energy was often reflected in his lectures—he once

[his:bio:tar:
sec](https://www.wikidata.org/wiki/Alfred_Tarski)

set fire to a wastebasket while disposing of a cigarette during a lecture, and was forbidden from lecturing in that building again.

Tarski had a thirst for knowledge from a young age. Although later in life he would tell students that he studied logic because it was the only class in which he got a B, his high school records show that he got A's across the board—even in logic. He studied at the University of Warsaw from 1918 to 1924. Tarski first intended to study biology, but became interested in mathematics, philosophy, and logic, as the university was the center of the Warsaw School of Logic and Philosophy. Tarski earned his doctorate in 1924 under the supervision of Stanisław Leśniewski.



Figure 1.9: Alfred Tarski

Before emigrating to the United States in 1939, Tarski completed some of his most important work while working as a secondary school teacher in Warsaw. His work on logical consequence and logical truth were written during this time. In 1939, Tarski was visiting the United States for a lecture tour. During his visit, Germany invaded Poland, and because of his Jewish heritage, Tarski could not return. His wife and children remained in Poland until the end of the war, but were then able to emigrate to the United States as well. Tarski taught at Harvard, the College of the City of New York, and the Institute for Advanced Study at Princeton, and finally the University of California, Berkeley. There he founded the multidisciplinary program in Logic and the Methodology of Science. Tarski died on October 26, 1983 at the age of 82.

Further Reading For more on Tarski's life, see the biography *Alfred Tarski: Life and Logic* (Feferman and Feferman, 2004). Tarski's seminal works on logical consequence and truth are available in English in (Corcoran, 1983). All of Tarski's original works have been collected into a four volume series, (Tarski, 1981).

1.10 Alan Turing

his:bio:tur:
sec

Alan Turing was born in Mailda Vale, London, on June 23, 1912. He is considered the father of theoretical computer science. Turing's interest in the physical sciences and mathematics started at a young age. However, as a boy his interests were not represented well in his schools, where emphasis was placed on literature and classics. Consequently, he did poorly in school and was reprimanded by many of his teachers.

Turing attended King's College, Cambridge as an undergraduate, where he studied mathematics. In 1936 Turing developed (what is now called) the Turing machine as an attempt to precisely define the notion of a computable function and to prove the undecidability of the decision problem. He was beaten to the result by Alonzo Church, who proved the result via his own lambda calculus. Turing's paper was still published with reference to Church's result. Church invited Turing to Princeton, where he spent 1936–1938, and obtained a doctorate under Church.



Figure 1.10: Alan Turing

Despite his interest in logic, Turing's earlier interests in physical sciences remained prevalent. His practical skills were put to work during his service with the British cryptanalytic department at Bletchley Park during World War II. Turing was a central figure in cracking the cypher used by German Naval communications—the Enigma code. Turing's expertise in statistics and cryptography, together with the introduction of electronic machinery, gave the team the ability to crack the code by creating a de-crypting machine called a “bombe.” His ideas also helped in the creation of the world's first programmable electronic computer, the Colossus, also used at Bletchley park to break the German Lorenz cypher.

Turing was gay. Nevertheless, in 1942 he proposed to Joan Clarke, one of his teammates at Bletchley Park, but later broke off the engagement and confessed to her that he was homosexual. He had several lovers throughout his lifetime, although homosexual acts were then criminal offences in the UK. In 1952, Turing's house was burgled by a friend of his lover at the time, and when filing a police report, Turing admitted to having a homosexual relationship, under the impression that the government was on their way to legalizing homosexual acts. This was not true, and he was charged with gross indecency. Instead of going to prison, Turing opted for a hormone treatment that reduced libido. Turing was found dead on June 8, 1954, of a cyanide overdose—most likely suicide. He was given a royal pardon by Queen Elizabeth II in 2013.

Further Reading For a comprehensive biography of Alan Turing, see [Hodges \(2014\)](#). Turing's life and work inspired a play, *Breaking the Code*, which was produced in 1996 for TV starring Derek Jacobi as Turing. *The Imitation Game*, an Academy Award nominated film starring Benedict Cumberbatch and Kiera Knightley, is also loosely based on Alan Turing's life and time at Bletchley Park ([Tyldum, 2014](#)).

[Radiolab \(2012\)](#) has several podcasts on Turing’s life and work. BBC Horizon’s documentary *The Strange Life and Death of Dr. Turing* is available to watch online ([Sykes, 1992](#)). ([Theelen, 2012](#)) is a short video of a working LEGO Turing Machine—made to honour Turing’s centenary in 2012.

Turing’s original paper on Turing machines and the decision problem is [Turing \(1937\)](#).

1.11 Ernst Zermelo

[his:bio:zer:
sec](#)

Ernst Zermelo was born on July 27, 1871 in Berlin, Germany. He had five sisters, though his family suffered from poor health and only three survived to adulthood. His parents also passed away when he was young, leaving him and his siblings orphans when he was seventeen. Zermelo had a deep interest in the arts, and especially in poetry. He was known for being sharp, witty, and critical. His most celebrated mathematical achievements include the introduction of the axiom of choice (in 1904), and his axiomatization of set theory (in 1908).

Zermelo’s interests at university were varied. He took courses in physics, mathematics, and philosophy. Under the supervision of Hermann Schwarz, Zermelo completed his dissertation *Investigations in the Calculus of Variations* in 1894 at the University of Berlin. In 1897, he decided to pursue more studies at the University of Göttingen, where he was heavily influenced by the foundational work of David Hilbert. In 1899 he became eligible for professorship, but did not get one until eleven years later—possibly due to his strange demeanour and “nervous haste.”

Zermelo finally received a paid professorship at the University of Zurich in 1910, but was forced to retire in 1916 due to tuberculosis. After his recovery, he was given an honorary professorship at the University of Freiburg in 1921. During this time he worked on foundational mathematics. He became irritated with the works of Thoralf Skolem and Kurt Gödel, and publicly criticized their approaches in his papers. He was dismissed from his position at Freiburg in 1935, due to his unpopularity and his opposition to Hitler’s rise to power in Germany.

The later years of Zermelo’s life were marked by isolation. After his dismissal in 1935, he abandoned mathematics. He moved to the country where he lived modestly. He married in 1944, and became completely dependent on



Figure 1.11: Ernst Zermelo

his wife as he was going blind. Zermelo lost his sight completely by 1951. He passed away in Günterstal, Germany, on May 21, 1953.

Further Reading For a full biography of Zermelo, see [Ebbinghaus \(2015\)](#). Zermelo’s seminal 1904 and 1908 papers are available to read in the original German ([Zermelo, 1904, 1908](#)). Zermelo’s collected works, including his writing on physics, are available in English translation in ([Ebbinghaus et al., 2010](#); [Ebbinghaus and Kanamori, 2013](#)).

Photo Credits

Georg Cantor, p. 3: Portrait of Georg Cantor by Otto Zeth courtesy of the Universitätsarchiv, Martin-Luther Universität Halle–Wittenberg. UAHW Rep. 40-VI, Nr. 3 Bild 102.

Alonzo Church, p. 4: Portrait of Alonzo Church, undated, photographer unknown. Alonzo Church Papers; 1924–1995, (C0948) Box 60, Folder 3. Manuscripts Division, Department of Rare Books and Special Collections, Princeton University Library. © Princeton University. The Open Logic Project has obtained permission to use this image for inclusion in non-commercial OLP-derived materials. Permission from Princeton University is required for any other use.

Gerhard Gentzen, p. 5: Portrait of Gerhard Gentzen playing ping-pong courtesy of Ekhart Mentzler-Trott.

Kurt Gödel, p. 6: Portrait of Kurt Gödel, ca. 1925, photographer unknown. From the Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA, on deposit at Princeton University Library, Manuscript Division, Department of Rare Books and Special Collections, Kurt Gödel Papers, (C0282), Box 14b, #110000. The Open Logic Project has obtained permission from the Institute’s Archives Center to use this image for inclusion in non-commercial OLP-derived materials. Permission from the Archives Center is required for any other use.

Emmy Noether, p. 7: Portrait of Emmy Noether, ca. 1922, courtesy of the Abteilung für Handschriften und Seltene Drucke, Niedersächsische Staats- und Universitätsbibliothek Göttingen, Cod. Ms. D. Hilbert 754, Bl. 14 Nr. 73. Restored from an original scan by Joel Fuller.

Rózsa Péter, p. 9: Portrait of Rózsa Péter, undated, photographer unknown. Courtesy of Béla Andrásfai.

Julia Robinson, p. 10: Portrait of Julia Robinson, unknown photographer, courtesy of Neil D. Reid. The Open Logic Project has obtained permission to use this image for inclusion in non-commercial OLP-derived materials. Per-

mission is required for any other use.

Bertrand Russell, p. 12: Portrait of Bertrand Russell, ca. 1907, courtesy of the William Ready Division of Archives and Research Collections, McMaster University Library. Bertrand Russell Archives, Box 2, f. 4.

Alfred Tarski, p. 13: Passport photo of Alfred Tarski, 1939. Cropped and restored from a scan of Tarski's passport by Joel Fuller. Original courtesy of Bancroft Library, University of California, Berkeley. Alfred Tarski Papers, Banc MSS 84/49. The Open Logic Project has obtained permission to use this image for inclusion in non-commercial OLP-derived materials. Permission from Bancroft Library is required for any other use.

Alan Turing, p. 14: Portrait of Alan Mathison Turing by Elliott & Fry, 29 March 1951, NPG x82217, © National Portrait Gallery, London. Used under a Creative Commons BY-NC-ND 3.0 license.

Ernst Zermelo, p. 15: Portrait of Ernst Zermelo, ca. 1922, courtesy of the Abteilung für Handschriften und Seltene Drucke, Niedersächsische Staats- und Universitätsbibliothek Göttingen, Cod. Ms. D. Hilbert 754, Bl. 6 Nr. 25.

Bibliography

- Andrásfai, Béla. 1986. Rózsa (Rosa) Péter. *Periodica Polytechnica Electrical Engineering* 30(2-3): 139–145. URL <http://www.pp.bme.hu/ee/article/view/4651>.
- Aspray, William. 1984. The Princeton mathematics community in the 1930s: Alonzo Church. URL http://www.princeton.edu/mudd/finding_aids/mathoral/pmc05.htm. Interview.
- Baaz, Matthias, Christos H. Papadimitriou, Hilary W. Putnam, Dana S. Scott, and Charles L. Harper Jr. 2011. *Kurt Gödel and the Foundations of Mathematics: Horizons of Truth*. Cambridge: Cambridge University Press.
- Church, Alonzo. 1936a. A note on the Entscheidungsproblem. *Journal of Symbolic Logic* 1: 40–41.
- Church, Alonzo. 1936b. An unsolvable problem of elementary number theory. *American Journal of Mathematics* 58: 345–363.
- Corcoran, John. 1983. *Logic, Semantics, Metamathematics*. Indianapolis: Hackett, 2nd ed.
- Csicsery, George. 2016. Zala films: Julia Robinson and Hilbert’s tenth problem. URL <http://www.zalafilms.com/films/juliarobinson.html>.
- Dauben, Joseph. 1990. *Georg Cantor: His Mathematics and Philosophy of the Infinite*. Princeton: Princeton University Press.
- Davis, Martin, Hilary Putnam, and Julia Robinson. 1961. The decision problem for exponential Diophantine equations. *Annals of Mathematics* 74(3): 425–436. URL <http://www.jstor.org/stable/1970289>.
- Dick, Auguste. 1981. *Emmy Noether 1882–1935*. Boston: Birkhäuser.
- du Sautoy, Marcus. 2014. A brief history of mathematics: Georg Cantor. URL <http://www.bbc.co.uk/programmes/b00ss1j0>. Audio Recording.
- Duncan, Arlene. 2015. The Bertrand Russell Research Centre. URL <http://russell.mcmaster.ca/>.

- Ebbinghaus, Heinz-Dieter. 2015. *Ernst Zermelo: An Approach to his Life and Work*. Berlin: Springer-Verlag.
- Ebbinghaus, Heinz-Dieter, Craig G. Fraser, and Akihiro Kanamori. 2010. *Ernst Zermelo. Collected Works*, vol. 1. Berlin: Springer-Verlag.
- Ebbinghaus, Heinz-Dieter and Akihiro Kanamori. 2013. *Ernst Zermelo: Collected Works*, vol. 2. Berlin: Springer-Verlag.
- Enderton, Herbert B. forthcoming. Alonzo Church: Life and Work. In *The Collected Works of Alonzo Church*. Cambridge: MIT Press.
- Feferman, Anita and Solomon Feferman. 2004. *Alfred Tarski: Life and Logic*. Cambridge: Cambridge University Press.
- Feferman, Solomon. 1994. Julia Bowman Robinson 1919–1985. *Biographical Memoirs of the National Academy of Sciences* 63: 1–28. URL <http://www.nasonline.org/publications/biographical-memoirs/memoir-pdfs/robinson-julia.pdf>.
- Feferman, Solomon, John W. Dawson Jr., Stephen C. Kleene, Gregory H. Moore, Robert M. Solovay, and Jean van Heijenoort. 1986. *Kurt Gödel: Collected Works. Vol. 1: Publications 1929–1936*. Oxford: Oxford University Press.
- Feferman, Solomon, John W. Dawson Jr., Stephen C. Kleene, Gregory H. Moore, Robert M. Solovay, and Jean van Heijenoort. 1990. *Kurt Gödel: Collected Works. Vol. 2: Publications 1938–1974*. Oxford: Oxford University Press.
- Frey, Holly and Tracy V. Wilson. 2015. Stuff you missed in history class: Emmy Noether, mathematics trailblazer. URL <http://www.missedinhistory.com/podcasts/emmy-noether-mathematics-trailblazer/>. Podcast audio.
- Gentzen, Gerhard. 1935a. Untersuchungen über das logische Schließen I. *Mathematische Zeitschrift* 39: 176–210. English translation in Szabo (1969), pp. 68–131.
- Gentzen, Gerhard. 1935b. Untersuchungen über das logische Schließen II. *Mathematische Zeitschrift* 39: 176–210, 405–431. English translation in Szabo (1969), pp. 68–131.
- Gödel, Kurt. 1929. Über die Vollständigkeit des Logikkalküls [On the completeness of the calculus of logic]. Dissertation, Universität Wien. Reprinted and translated in Feferman et al. (1986), pp. 60–101.
- Gödel, Kurt. 1931. über formal unentscheidbare Sätze der *Principia Mathematica* und verwandter Systeme I [On formally undecidable propositions of *Principia Mathematica* and related systems I]. *Monatshefte für Mathematik*

- und *Physik* 38: 173–198. Reprinted and translated in [Feferman et al. \(1986\)](#), pp. 144–195.
- Grattan-Guinness, Ivor. 1971. Towards a biography of Georg Cantor. *Annals of Science* 27(4): 345–391.
- Hodges, Andrew. 2014. *Alan Turing: The Enigma*. London: Vintage.
- Institute, Perimeter. 2015. Emmy Noether: Her life, work, and influence. URL <https://www.youtube.com/watch?v=tNNyAyMRsgE>. Video Lecture.
- Irvine, Andrew David. 2015. Sound clips of Bertrand Russell speaking. URL <http://plato.stanford.edu/entries/russell/russell-soundclips.html>.
- Jacobson, Nathan. 1983. *Emmy Noether: Gesammelte Abhandlungen—Collected Papers*. Berlin: Springer-Verlag.
- John Dawson, Jr. 1997. *Logical Dilemmas: The Life and Work of Kurt Gödel*. Boca Raton: CRC Press.
- LibriVox. n.d. Bertrand Russell. URL https://librivox.org/author/1508?primary_key=1508&search_category=author&search_page=1&search_form=get_results. Collection of public domain audiobooks.
- Linsenmayer, Mark. 2014. The partially examined life: Gödel on math. URL <http://www.partiallyexaminedlife.com/2014/06/16/ep95-godel/>. Podcast audio.
- MacFarlane, John. 2015. Alonzo Church’s JSL reviews. URL <http://johnmacfarlane.net/church.html>.
- Matijasevich, Yuri. 1992. My collaboration with Julia Robinson. *The Mathematical Intelligencer* 14(4): 38–45.
- Menzler-Trott, Eckart. 2007. *Logic’s Lost Genius: The Life of Gerhard Gentzen*. Providence: American Mathematical Society.
- O’Connor, John J. and Edmund F. Robertson. 2014. Rózsa Péter. URL <http://www-groups.dcs.st-and.ac.uk/~history/Biographies/Peter.html>.
- Péter, Rózsa. 1935a. Über den Zusammenhang der verschiedenen Begriffe der rekursiven Funktion. *Mathematische Annalen* 110: 612–632.
- Péter, Rózsa. 1935b. Konstruktion nichtrekursiver Funktionen. *Mathematische Annalen* 111: 42–60.
- Péter, Rózsa. 1951. *Rekursive Funktionen*. Budapest: Akadémiai Kiado. English translation in ([Péter, 1967](#)).
- Péter, Rózsa. 1967. *Recursive Functions*. New York: Academic Press.

- Péter, Rózsa. 2010. *Playing with Infinity*. New York: Dover.
URL https://books.google.ca/books?id=6V3wNs4uv_4C&lpg=PP1&ots=BkQZaHcR99&lr&pg=PP1#v=onepage&q&f=false.
- Radiolab. 2012. The Turing problem. URL <http://www.radiolab.org/story/193037-turing-problem/>. Podcast audio.
- Reid, Constance. 1986. The autobiography of Julia Robinson. *The College Mathematics Journal* 17: 3–21.
- Reid, Constance. 1996. *Julia: A Life in Mathematics*. Cambridge: Cambridge University Press. URL <https://books.google.ca/books?id=1RtSzQyHf9UC&lpg=PP1&pg=PP1#v=onepage&q&f=false>.
- Robinson, Julia. 1949. Definability and decision problems in arithmetic. *Journal of Symbolic Logic* 14(2): 98–114. URL <http://www.jstor.org/stable/2266510>.
- Robinson, Julia. 1996. *The Collected Works of Julia Robinson*. Providence: American Mathematical Society.
- Rose, Daniel. 2012. A song about Georg Cantor. URL <https://www.youtube.com/watch?v=QUP5Z4Fb5k4>. Audio Recording.
- Russell, Bertrand. 1905. On denoting. *Mind* 14: 479–493.
- Russell, Bertrand. 1967. *The Autobiography of Bertrand Russell*, vol. 1. London: Allen and Unwin.
- Russell, Bertrand. 1968. *The Autobiography of Bertrand Russell*, vol. 2. London: Allen and Unwin.
- Russell, Bertrand. 1969. *The Autobiography of Bertrand Russell*, vol. 3. London: Allen and Unwin.
- Russell, Bertrand. n.d. Bertrand Russell on smoking. URL https://www.youtube.com/watch?v=80oLTiVW_1c. Video Interview.
- Segal, Sanford L. 2014. *Mathematicians under the Nazis*. Princeton: Princeton University Press.
- Sigmund, Karl, John Dawson, Kurt Mühlberger, Hans Magnus Enzensberger, and Juliette Kennedy. 2007. Kurt Gödel: Das Album–The Album. *The Mathematical Intelligencer* 29(3): 73–76.
- Smith, Peter. 2013. *An Introduction to Gödel's Theorems*. Cambridge: Cambridge University Press.
- Sykes, Christopher. 1992. BBC Horizon: The strange life and death of Dr. Turing. URL <https://www.youtube.com/watch?v=gyusnGbBSHE>.

- Szabo, Manfred E. 1969. *The Collected Papers of Gerhard Gentzen*. Amsterdam: North-Holland.
- Takeuti, Gaisi, Nicholas Passell, and Mariko Yasugi. 2003. *Memoirs of a Proof Theorist: Gödel and Other Logicians*. Singapore: World Scientific.
- Tamassy, Istvan. 1994. Interview with Róza Péter. *Modern Logic* 4(3): 277–280.
- Tarski, Alfred. 1981. *The Collected Works of Alfred Tarski*, vol. I–IV. Basel: Birkhäuser.
- Theelen, Andre. 2012. Lego turing machine. URL <https://www.youtube.com/watch?v=FTSAiF9AHN4>.
- Turing, Alan M. 1937. On computable numbers, with an application to the “Entscheidungsproblem”. *Proceedings of the London Mathematical Society, 2nd Series* 42: 230–265.
- Tyldum, Morten. 2014. The imitation game. Motion picture.
- Wang, Hao. 1990. *Reflections on Kurt Gödel*. Cambridge: MIT Press.
- Zermelo, Ernst. 1904. Beweis, daß jede Menge wohlgeordnet werden kann. *Mathematische Annalen* 59: 514–516. English translation in (Ebbinghaus et al., 2010, pp. 115–119).
- Zermelo, Ernst. 1908. Untersuchungen über die Grundlagen der Mengenlehre I. *Mathematische Annalen* 65(2): 261–281. English translation in (Ebbinghaus et al., 2010, pp. 189–229).