tab.1 Soundness with Identity predicate

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Proposition tab.1. *Tableaux with rules for identity are sound: no closed tableau is satisfiable.*

Proof. We just have to show as before that if a tableau has a satisfiable branch, the branch resulting from applying one of the rules for $=$ to it is also satisfiable. Let Γ be the set of signed formulas on the branch, and let $\mathfrak{M}$ be a structure satisfying Γ.

Suppose the branch is expanded using $=$, i.e., by adding the signed formula $\mathbb{T}\, t = t$. Trivially, $\mathfrak{M} \vDash t = t$, so $\mathfrak{M}$ also satisfies $\Gamma \cup \{\mathbb{T}\, t = t\}$.

If the branch is expanded using $=\mathbb{T}$, we add a signed formula $S\,\varphi(t_2)$, but Γ contains both $\mathbb{T}\, t_1 = t_2$ and $\mathbb{T}\,\varphi(t_1)$. Thus we have $\mathfrak{M} \vDash t_1 = t_2$ and $\mathfrak{M} \vDash \varphi(t_1)$. Let s be a variable assignment with $s(x) = \mathrm{Val}^{\mathfrak{M}}(t_1)$. By **??**, $\mathfrak{M}, s \vDash \varphi(t_1)$. Since $s \sim_x s$, by **??**, $\mathfrak{M}, s \vDash \varphi(x)$. since $\mathfrak{M} \vDash t_1 = t_2$, we have $\mathrm{Val}^{\mathfrak{M}}(t_1) = \mathrm{Val}^{\mathfrak{M}}(t_2)$, and hence $s(x) = \mathrm{Val}^{\mathfrak{M}}(t_2)$. By applying **??** again, we also have $\mathfrak{M}, s \vDash \varphi(t_2)$. By **??**, $\mathfrak{M} \vDash \varphi(t_2)$. The case of $=\mathbb{F}$ is treated similarly. □

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Bibliography