A tableau is a systematic survey of the possible ways a sentence can be true or false in a structure. The building blocks of a tableau are signed formulas: sentences plus a truth value “sign,” either $T$ or $F$. These signed formulas are arranged in a (downward growing) tree.

**Definition tab.1.** A *signed formula* is a pair consisting of a truth value and a sentence, i.e., either:

$$T\varphi \text{ or } F\varphi.$$ 

Intuitively, we might read $T\varphi$ as “$\varphi$ might be true” and $F\varphi$ as “$\varphi$ might be false” (in some structure).

Each signed formula in the tree is either an assumption (which are listed at the very top of the tree), or it is obtained from a signed formula above it by one of a number of rules of inference. There are two rules for each possible main operator of the preceding formula, one for the case when the sign is $T$, and one for the case where the sign is $F$. Some rules allow the tree to branch, and some only add signed formulas to the branch. A rule may be (and often must be) applied not to the immediately preceding signed formula, but to any signed formula in the branch from the root to the place the rule is applied.

A branch is *closed* when it contains both $T\varphi$ and $F\varphi$. A closed tableau is one where every branch is closed. Under the intuitive interpretation, any branch describes a joint possibility, but $T\varphi$ and $F\varphi$ are not jointly possible. In other words, if a branch is closed, the possibility it describes has been ruled out. In particular, that means that a closed tableau rules out all possibilities of simultaneously making every assumption of the form $T\varphi$ true and every assumption of the form $F\varphi$ false.

A closed tableau for $\varphi$ is a closed tableau with root $F\varphi$. If such a closed tableau exists, all possibilities for $\varphi$ being false have been ruled out; i.e., $\varphi$ must be true in every structure.

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**Bibliography**