

## tab.1 Rules and Tableaux

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sec A **tableau** is a systematic survey of the possible ways a **sentence** can be true or false in a **structure**. The building blocks of a tableau are **signed formulas: sentences** plus a truth value “sign,” either  $\mathbb{T}$  or  $\mathbb{F}$ . These signed **formulas** are arranged in a (downward growing) tree.

**Definition tab.1.** A *signed formula* is a pair consisting of a truth value and a **sentence**, i.e., either:

$$\mathbb{T}\varphi \text{ or } \mathbb{F}\varphi.$$

Intuitively, we might read  $\mathbb{T}\varphi$  as “ $\varphi$  might be true” and  $\mathbb{F}\varphi$  as “ $\varphi$  might be false” (in some **structure**).

Each **signed formula** in the tree is either an *assumption* (which are listed at the very top of the tree), or it is obtained from a **signed formula** above it by one of a number of rules of inference. There are two rules for each possible **main operator** of the preceding **formula**, one for the case when the sign is  $\mathbb{T}$ , and one for the case where the sign is  $\mathbb{F}$ . Some rules allow the tree to branch, and some only add **signed formulas** to the branch. A rule may be (and often must be) applied not to the immediately preceding **signed formula**, but to any **signed formula** in the branch from the root to the place the rule is applied.

A branch is *closed* when it contains both  $\mathbb{T}\varphi$  and  $\mathbb{F}\varphi$ . A closed **tableau** is one where every branch is closed. Under the intuitive interpretation, any branch describes a joint possibility, but  $\mathbb{T}\varphi$  and  $\mathbb{F}\varphi$  are not jointly possible. In other words, if a branch is closed, the possibility it describes has been ruled out. In particular, that means that a closed **tableau** rules out all possibilities of simultaneously making every assumption of the form  $\mathbb{T}\varphi$  true and every assumption of the form  $\mathbb{F}\varphi$  false.

A closed **tableau for**  $\varphi$  is a closed **tableau** with root  $\mathbb{F}\varphi$ . If such a closed **tableau** exists, all possibilities for  $\varphi$  being false have been ruled out; i.e.,  $\varphi$  must be true in every **structure**.

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## Bibliography