

**tab.1  Quantifier Rules**

**Rules for \( \forall \)**

<table>
<thead>
<tr>
<th>( \frac{T \forall x \varphi(x)}{T \varphi(t)} ) ( \forall T )</th>
<th>( \frac{F \forall x \varphi(x)}{F \varphi(a)} ) ( \forall F )</th>
</tr>
</thead>
</table>

In \( \forall T \), \( t \) is a closed term (i.e., one without variables). In \( \forall F \), \( a \) is a constant symbol which must not occur anywhere in the branch above \( \forall F \) rule. We call \( a \) the eigenvariable of the \( \forall F \) inference.

**Rules for \( \exists \)**

<table>
<thead>
<tr>
<th>( \frac{T \exists x \varphi(x)}{T \varphi(a)} ) ( \exists T )</th>
<th>( \frac{F \exists x \varphi(x)}{F \varphi(t)} ) ( \exists F )</th>
</tr>
</thead>
</table>

Again, \( t \) is a closed term, and \( a \) is a constant symbol which does not occur in the branch above the \( \exists F \) rule. We call \( a \) the eigenvariable of the \( \exists F \) inference.

We use the term “eigenvariable” even though \( a \) in the above rules is a constant symbol. This has historical reasons.

In \( \forall T \) and \( \exists F \) there are no restrictions on the term \( t \). On the other hand, in the \( \exists T \) and \( \forall F \) rules, the eigenvariable condition requires that the constant symbol \( a \) does not occur anywhere in the branches above the respective inference. It is necessary to ensure that the system is sound. Without this condition, the following would be a closed tableau for \( \exists x \varphi(x) \rightarrow \forall x \varphi(x) \):

1. \( F \exists x \varphi(x) \rightarrow \forall x \varphi(x) \) Assumption
2. \( T \exists x \varphi(x) \rightarrow F 1 \)
3. \( F \forall x \varphi(x) \rightarrow F 1 \)
4. \( T \varphi(a) \rightarrow \exists T 2 \)
5. \( F \varphi(a) \rightarrow \forall F 3 \)

However, \( \exists x \varphi(x) \rightarrow \forall x \varphi(x) \) is not valid.
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Bibliography