tab.1 Quantifier Rules

fol:tab:qrl:sec

Rules for $\forall$

$$\frac{\mathbb{T}\,\forall x\,\varphi(x)}{\mathbb{T}\,\varphi(t)}\;\forall\mathbb{T} \qquad\qquad \frac{\mathbb{F}\,\forall x\,\varphi(x)}{\mathbb{F}\,\varphi(a)}\;\forall\mathbb{F}$$

In $\forall\mathbb{T}$, t is a closed term (i.e., one without variables). In $\forall\mathbb{F}$, a is a constant symbol which must not occur anywhere in the branch above $\forall\mathbb{F}$rule. We call a the *eigenvariable* of the $\forall\mathbb{F}$inference.

Rules for $\exists$

$$\frac{\mathbb{T}\,\exists x\,\varphi(x)}{\mathbb{T}\,\varphi(a)}\;\exists\mathbb{T} \qquad\qquad \frac{\mathbb{F}\,\exists x\,\varphi(x)}{\mathbb{F}\,\varphi(t)}\;\exists\mathbb{F}$$

Again, t is a closed term, and a is a constant symbol which does not occur in the branch above the $\exists\mathbb{F}$rule. We call a the *eigenvariable* of the $\exists\mathbb{F}$inference.

The condition that an eigenvariable not occur in the branch above the $\forall\mathbb{F}$or $\exists\mathbb{T}$inference is called the *eigenvariable condition*.

We use the term "eigenvariable" even though a in the above rules is a constant symbol. This has historical reasons. explanation

In $\forall\mathbb{T}$and $\exists\mathbb{F}$there are no restrictions on the term t. On the other hand, in the $\exists\mathbb{T}$and $\forall\mathbb{F}$rules, the eigenvariable condition requires that the constant symbol a does not occur anywhere in the branches above the respective inference. It is necessary to ensure that the system is sound. Without this condition, the following would be a closed tableau for $\exists x\,\varphi(x) \to \forall x\,\varphi(x)$:

1.	$\mathbb{F}\,\exists x\,\varphi(x) \to \forall x\,\varphi(x)$	Assumption
2.	$\mathbb{T}\,\exists x\,\varphi(x)$	$\to\mathbb{F}\,1$
3.	$\mathbb{F}\,\forall x\,\varphi(x)$	$\to\mathbb{F}\,1$
4.	$\mathbb{T}\,\varphi(a)$	$\exists\mathbb{T}\,2$
5.	$\mathbb{F}\,\varphi(a)$	$\forall\mathbb{F}\,3$
	$\otimes$	

However, $\exists x\,\varphi(x) \to \forall x\,\varphi(x)$ is not valid.

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Bibliography