tab.1 Quantifier Rules

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Rules for $\forall$

$$\frac{\mathbb{T}\,\forall x\,\varphi(x)}{\mathbb{T}\,\varphi(t)}\,\forall\mathbb{T} \qquad\qquad \frac{\mathbb{F}\,\forall x\,\varphi(x)}{\mathbb{F}\,\varphi(a)}\,\forall\mathbb{F}$$

In $\forall\mathbb{T}$, t is a closed term (i.e., one without variables). In $\forall\mathbb{F}$, a is a constant symbol which must not occur anywhere in the branch above $\forall\mathbb{F}$ rule. We call a the *eigenvariable* of the $\forall\mathbb{F}$ inference.[1]

Rules for $\exists$

$$\frac{\mathbb{T}\,\exists x\,\varphi(x)}{\mathbb{T}\,\varphi(a)}\,\exists\mathbb{T} \qquad\qquad \frac{\mathbb{F}\,\exists x\,\varphi(x)}{\mathbb{F}\,\varphi(t)}\,\exists\mathbb{F}$$

Again, t is a closed term, and a is a constant symbol which does not occur in the branch above the $\exists\mathbb{T}$ rule. We call a the *eigenvariable* of the $\exists\mathbb{T}$ inference.

The condition that an eigenvariable not occur in the branch above the $\forall\mathbb{F}$ or $\exists\mathbb{T}$ inference is called the *eigenvariable condition.*

Recall the convention that when φ is a formula with the variable x free, we indicate this by writing $\varphi(x)$. In the same context, $\varphi(t)$ then is short for $\varphi[t/x]$. So we could also write the $\exists\mathbb{F}$ rule as: explanation

$$\frac{\mathbb{F}\,\exists x\,\varphi}{\mathbb{F}\,\varphi[t/x]}\,\exists\mathbb{F}$$

Note that t may already occur in φ, e.g., φ might be $P(t, x)$. Thus, inferring $\mathbb{F}\,P(t, t)$ from $\mathbb{F}\,\exists x\,P(t, x)$ is a correct application of $\exists\mathbb{F}$. However, the eigenvariable conditions in $\forall\mathbb{F}$ and $\exists\mathbb{T}$ require that the constant symbol a does not occur in φ. So, you cannot correctly infer $\mathbb{F}\,P(a, a)$ from $\mathbb{F}\,\forall x\,P(a, x)$ using $\forall\mathbb{F}$.

In $\forall\mathbb{T}$ and $\exists\mathbb{F}$ there are no restrictions on the term t. On the other hand, in the $\exists\mathbb{T}$ and $\forall\mathbb{F}$ rules, the eigenvariable condition requires that the constant symbol a does not occur anywhere in the branches above the respective inference. It is necessary to ensure that the system is sound. Without this condition, the following would be a closed tableau for $\exists x\,\varphi(x) \to \forall x\,\varphi(x)$: explanation

[1] We use the term "eigenvariable" even though a in the above rule is a constant symbol. This has historical reasons.

1.	$\mathbb{F}\,\exists x\,\varphi(x) \to \forall x\,\varphi(x)$	Assumption
2.	$\mathbb{T}\,\exists x\,\varphi(x)$	$\to\mathbb{F}\,1$
3.	$\mathbb{F}\,\forall x\,\varphi(x)$	$\to\mathbb{F}\,1$
4.	$\mathbb{T}\,\varphi(a)$	$\exists\mathbb{T}\,2$
5.	$\mathbb{F}\,\varphi(a)$	$\forall\mathbb{F}\,3$
	$\otimes$	

However, $\exists x\,\varphi(x) \to \forall x\,\varphi(x)$ is not valid.

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Bibliography