tab.1  Quantifier Rules

Rules for $\forall$

\[
\begin{array}{c}
\frac{T \forall x \varphi(x)}{T \varphi(t)} \ \forall T \\
\frac{F \forall x \varphi(x)}{F \varphi(a)} \ \forall F
\end{array}
\]

In $\forall T$, $t$ is a closed term (i.e., one without variables). In $\forall F$, $a$ is a constant symbol which must not occur anywhere in the branch above $\forall F$ rule. We call $a$ the eigenvariable of the $\forall F$ inference.

Rules for $\exists$

\[
\begin{array}{c}
\frac{T \exists x \varphi(x)}{T \varphi(a)} \ \exists T \\
\frac{F \exists x \varphi(x)}{F \varphi(t)} \ \exists F
\end{array}
\]

Again, $t$ is a closed term, and $a$ is a constant symbol which does not occur in the branch above the $\exists F$ rule. We call $a$ the eigenvariable of the $\exists F$ inference.

The condition that an eigenvariable not occur in the branch above the $\forall F$ or $\exists T$ inference is called the eigenvariable condition.

We use the term “eigenvariable” even though $a$ in the above rules is a constant symbol. This has historical reasons.

In $\forall T$ and $\exists F$ there are no restrictions on the term $t$. On the other hand, in the $\exists T$ and $\forall F$ rules, the eigenvariable condition requires that the constant symbol $a$ does not occur anywhere in the branches above the respective inference. It is necessary to ensure that the system is sound. Without this condition, the following would be a closed tableau for $\exists x \varphi(x) \rightarrow \forall x \varphi(x)$:

1. $F \exists x \varphi(x) \rightarrow \forall x \varphi(x)$ Assumption
2. $T \exists x \varphi(x) \rightarrow F 1$
3. $F \forall x \varphi(x) \rightarrow F 1$
4. $T \varphi(a) \rightarrow F 1$
5. $F \varphi(a) \rightarrow F 1$

However, $\exists x \varphi(x) \rightarrow \forall x \varphi(x)$ is not valid.