tab.1  Quantifier Rules

Rules for ∀

\[
\begin{array}{c}
\frac{T \forall x \varphi(x)}{T \varphi(t)} & \forall T \\
\frac{F \forall x \varphi(x)}{F \varphi(a)} & \forall F
\end{array}
\]

In ∀T, \(t\) is a closed term (i.e., one without variables). In ∀F, \(a\) is a constant symbol which must not occur anywhere in the branch above ∀F rule. We call \(a\) the eigenvariable of the ∀F inference.\(^1\)

Rules for ∃

\[
\begin{array}{c}
\frac{T \exists x \varphi(x)}{T \varphi(a)} & \exists T \\
\frac{F \exists x \varphi(x)}{F \varphi(t)} & \exists F
\end{array}
\]

Again, \(t\) is a closed term, and \(a\) is a constant symbol which does not occur in the branch above the ∃T rule. We call \(a\) the eigenvariable of the ∃T inference.

The condition that an eigenvariable not occur in the branch above the ∀F or ∃T inference is called the eigenvariable condition.

Recall the convention that when \(\varphi\) is a formula with the variable \(x\) free, we indicate this by writing \(\varphi(x)\). In the same context, \(\varphi(t)\) then is short for \(\varphi[t/x]\). So we could also write the ∃F rule as:

\[
\frac{F \exists x \varphi}{F \varphi[t/x]} \quad \exists F
\]

Note that \(t\) may already occur in \(\varphi\), e.g., \(\varphi\) might be \(P(t, x)\). Thus, inferring \(F P(t, t)\) from \(F \exists x P(t, x)\) is a correct application of ∃F. However, the eigenvariable conditions in ∀F and ∃T require that the constant symbol \(a\) does not occur in \(\varphi\). So, you cannot correctly infer \(F P(a, a)\) from \(F \forall x P(a, x)\) using ∀F.

In ∀T and ∃F there are no restrictions on the term \(t\). On the other hand, in the ∃T and ∀F rules, the eigenvariable condition requires that the constant symbol \(a\) does not occur anywhere in the branches above the respective inference. It is necessary to ensure that the system is sound. Without this condition, the following would be a closed tableau for \(\exists x \varphi(x) \rightarrow \forall x \varphi(x)\):

\[\text{We use the term “eigenvariable” even though \(a\) in the above rule is a constant symbol. This has historical reasons.}\]

\(^1\)
However, $\exists x \varphi(x) \rightarrow \forall x \varphi(x)$ is not valid.