tab.1 Derivability and the Quantifiers

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fol:tab:qpr: thm:strong-generalization

Theorem tab.1. *If c is a constant not occurring in Γ or $\varphi(x)$ and $\Gamma \vdash \varphi(c)$, then $\Gamma \vdash \forall x\, \varphi(x)$.*

Proof. Suppose $\Gamma \vdash \varphi(c)$, i.e., there are $\psi_1, \ldots, \psi_n \in \Gamma$ and a closed tableau for

$$\{\mathbb{F}\,\varphi(c), \mathbb{T}\,\psi_1, \ldots, \mathbb{T}\,\psi_n\}.$$

We have to show that there is also a closed tableau for

$$\{\mathbb{F}\,\forall x\,\varphi(x), \mathbb{T}\,\psi_1, \ldots, \mathbb{T}\,\psi_n\}.$$

Take the closed tableau and replace the first assumption with $\mathbb{F}\,\forall x\,\varphi(x)$, and insert $\mathbb{F}\,\varphi(c)$ after the assumptions.

$$\begin{array}{c} \mathbb{F}\,\varphi(c) \\ \mathbb{T}\,\psi_1 \\ \vdots \\ \mathbb{T}\,\psi_n \end{array} \qquad \begin{array}{c} \mathbb{F}\,\forall x\,\varphi(x) \\ \mathbb{T}\,\psi_1 \\ \vdots \\ \mathbb{T}\,\psi_n \\ \mathbb{F}\,\varphi(c) \end{array}$$

The tableau is still closed, since all sentences available as assumptions before are still available at the top of the tableau. The inserted line is the result of a correct application of $\forall\mathbb{F}$, since the constant symbol c does not occur in ψ_1, $\ldots$, ψ_n of $\forall x\,\varphi(x)$, i.e., it does not occur above the inserted line in the new tableau. □

fol:tab:qpr: prop:provability-quantifiers

Proposition tab.2.

1. $\varphi(t) \vdash \exists x\,\varphi(x)$.
2. $\forall x\,\varphi(x) \vdash \varphi(t)$.

Proof. 1. A closed tableau for $\mathbb{F}\,\exists x\,\varphi(x), \mathbb{T}\,\varphi(t)$ is:

$$\begin{array}{lcl} 1. & \mathbb{F}\,\exists x\,\varphi(x) & \text{Assumption} \\ 2. & \mathbb{T}\,\varphi(t) & \text{Assumption} \\ 3. & \mathbb{F}\,\varphi(t) & \exists\mathbb{F}\,1 \\ & \otimes & \end{array}$$

2. A closed tableau for $\mathbb{F}\,\varphi(t), \mathbb{T}\,\forall x\,\varphi(x)$, is:

1.	$\mathbb{F}\,\varphi(t)$	Assumption
2.	$\mathbb{T}\,\forall x\,\varphi(x)$	Assumption
3.	$\mathbb{T}\,\varphi(t)$	$\forall\mathbb{T}\,2$
	$\otimes$	

$\square$

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Bibliography