Theorem tab.1. If $c$ is a constant not occurring in $\Gamma$ or $\varphi(x)$ and $\Gamma \vdash \varphi(c)$, then $\Gamma \vdash \forall x \varphi(x)$.

Proof. Suppose $\Gamma \vdash \varphi(c)$, i.e., there are $\psi_1, \ldots, \psi_n \in \Gamma$ and a closed tableau for

$$\{ F \varphi(c), T \psi_1, \ldots, T \psi_n \}.$$  

We have to show that there is also a closed tableau for

$$\{ F \forall x \varphi(x), T \psi_1, \ldots, T \psi_n \}.$$  

Take the closed tableau and replace the first assumption with $F \forall x \varphi(x)$, and insert $F \varphi(c)$ after the assumptions.

$$
\begin{array}{c}
F \varphi(c) \\
T \psi_1 \\
\vdots \\
T \psi_n \\
\hline \\
F \forall x \varphi(x) \\
T \psi_1 \\
\vdots \\
T \psi_n \\
\hline \\
F \varphi(c)
\end{array}
$$

The tableau is still closed, since all sentences available as assumptions before are still available at the top of the tableau. The inserted line is the result of a correct application of $\forall F$, since the constant symbol $c$ does not occur in $\psi_1, \ldots, \psi_n$ of $\forall x \varphi(x)$, i.e., it does not occur above the inserted line in the new tableau.

Proposition tab.2.

1. $\varphi(t) \vdash \exists x \varphi(x)$.

2. $\forall x \varphi(x) \vdash \varphi(t)$.

Proof. 1. A closed tableau for $F \exists x \varphi(x), T \varphi(t)$ is:

$$
\begin{array}{c}
F \exists x \varphi(x) \\
T \varphi(t) \\
\hline \\
F \varphi(t) \ \exists F 1
\end{array}
$$

2. A closed tableau for $F \varphi(t), T \forall x \varphi(x)$ is:
1. $\mathcal{F}\varphi(t)$ Assumption
2. $T\forall x \varphi(x)$ Assumption
3. $T \varphi(t)$ $\forall T \ 2$

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Bibliography