

tab.1 Derivability and the Quantifiers

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thm:strong-generalization

Theorem tab.1. *If c is a constant not occurring in Γ or $\varphi(x)$ and $\Gamma \vdash \varphi(c)$, then $\Gamma \vdash \forall x \varphi(x)$.*

Proof. Suppose $\Gamma \vdash \varphi(c)$, i.e., there are $\psi_1, \dots, \psi_n \in \Gamma$ and a closed **tableau** for

$$\{\mathbb{F} \varphi(c), \mathbb{T} \psi_1, \dots, \mathbb{T} \psi_n\}.$$

We have to show that there is also a closed **tableau** for

$$\{\mathbb{F} \forall x \varphi(x), \mathbb{T} \psi_1, \dots, \mathbb{T} \psi_n\}.$$

Take the closed **tableau** and replace the first assumption with $\mathbb{F} \forall x \varphi(x)$, and insert $\mathbb{F} \varphi(c)$ after the assumptions.

$$\begin{array}{cc} \mathbb{F} \varphi(c) & \mathbb{F} \forall x \varphi(x) \\ \mathbb{T} \psi_1 & \mathbb{T} \psi_1 \\ \vdots & \vdots \\ \mathbb{T} \psi_n & \mathbb{T} \psi_n \\ \mathbb{F} \varphi(c) & \mathbb{F} \varphi(c) \end{array}$$

The tableau is still closed, since all **sentences** available as assumptions before are still available at the top of the **tableau**. The inserted line is the result of a correct application of $\forall\mathbb{F}$, since the **constant symbol** c does not occur in ψ_1, \dots, ψ_n of $\forall x \varphi(x)$, i.e., it does not occur above the inserted line in the new **tableau**. \square

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prop:provability-quantifiers

Proposition tab.2.

1. $\varphi(t) \vdash \exists x \varphi(x)$.
2. $\forall x \varphi(x) \vdash \varphi(t)$.

Proof. 1. A closed **tableau** for $\mathbb{F} \exists x \varphi(x), \mathbb{T} \varphi(t)$ is:

$$\begin{array}{ll} 1. & \mathbb{F} \exists x \varphi(x) \quad \text{Assumption} \\ 2. & \mathbb{T} \varphi(t) \quad \text{Assumption} \\ 3. & \mathbb{F} \varphi(t) \quad \exists\mathbb{F} 1 \\ & \otimes \end{array}$$

2. A closed **tableau** for $\mathbb{F} \varphi(t), \mathbb{T} \forall x \varphi(x)$, is:

1. $\mathbb{F} \varphi(t)$ Assumption
2. $\mathbb{T} \forall x \varphi(x)$ Assumption
3. $\mathbb{T} \varphi(t)$ $\forall \mathbb{T} 2$
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Bibliography