The completeness theorem also requires that the tableaux rules yield the facts about \( \vdash \) established in this section.

**Theorem tab.1.** If \( c \) is a constant not occurring in \( \Gamma \) or \( \varphi(x) \) and \( \Gamma \vdash \varphi(c) \), then \( \Gamma \vdash \forall x \varphi(x) \).

**Proof.** Suppose \( \Gamma \vdash \varphi(c) \), i.e., there are \( \psi_1, \ldots, \psi_n \in \Gamma \) and a closed tableau for

\[
\{ \neg \varphi(c), \top \psi_1, \ldots, \top \psi_n \}.
\]

We have to show that there is also a closed tableau for

\[
\{ \neg \forall x \varphi(x), \top \psi_1, \ldots, \top \psi_n \}.
\]

Take the closed tableau and replace the first assumption with \( \neg \forall x \varphi(x) \), and insert \( \neg \varphi(c) \) after the assumptions.

The tableau is still closed, since all sentences available as assumptions before are still available at the top of the tableau. The inserted line is the result of a correct application of \( \forall F \), since the constant symbol \( c \) does not occur in \( \psi_1, \ldots, \psi_n \) or \( \forall x \varphi(x) \), i.e., it does not occur above the inserted line in the new tableau. \( \square \)

**Proposition tab.2.**

1. \( \varphi(t) \vdash \exists x \varphi(x) \).
2. \( \forall x \varphi(x) \vdash \varphi(t) \).

**Proof.** 1. A closed tableau for \( \neg \exists x \varphi(x), \top \varphi(t) \) is:

\[
\begin{array}{c}
1. \neg \exists x \varphi(x) \\
2. \top \varphi(t) \\
3. \neg \exists F \\
\end{array}
\]

2. A closed tableau for \( \neg \varphi(t), \top \forall x \varphi(x) \), is:
1. $F \varphi(t)$ Assumption
2. $T \forall x \varphi(x)$ Assumption
3. $T \varphi(t) \otimes \forall T 2$

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Bibliography