

tab.1 Derivability and the Propositional Connectives

fol:tab:ppr:
sec

Proposition tab.1.

fol:tab:ppr:
prop:provability-land

fol:tab:ppr:
prop:provability-land-left

fol:tab:ppr:
prop:provability-land-right

1. Both $\varphi \wedge \psi \vdash \varphi$ and $\varphi \wedge \psi \vdash \psi$.
2. $\varphi, \psi \vdash \varphi \wedge \psi$.

Proof. 1. Both $\{\mathbb{F} \varphi, \mathbb{T} \varphi \wedge \psi\}$ and $\{\mathbb{F} \psi, \mathbb{T} \varphi \wedge \psi\}$ have closed **tableaux**

1.	$\mathbb{F} \varphi$	Assumption
2.	$\mathbb{T} \varphi \wedge \psi$	Assumption
3.	$\mathbb{T} \varphi$	$\wedge \mathbb{T} 2$
4.	$\mathbb{T} \psi$	$\wedge \mathbb{T} 2$
	\otimes	

1.	$\mathbb{F} \psi$	Assumption
2.	$\mathbb{T} \varphi \wedge \psi$	Assumption
3.	$\mathbb{T} \varphi$	$\wedge \mathbb{T} 2$
4.	$\mathbb{T} \psi$	$\wedge \mathbb{T} 2$
	\otimes	

2. Here is a closed **tableau** for $\{\mathbb{T} \varphi, \mathbb{T} \psi, \mathbb{F} \varphi \wedge \psi\}$:

1.	$\mathbb{F} \varphi \wedge \psi$	Assumption
2.	$\mathbb{T} \varphi$	Assumption
3.	$\mathbb{T} \psi$	Assumption
4.	$\begin{array}{c} \diagdown \quad \diagup \\ \mathbb{F} \varphi \quad \mathbb{F} \psi \\ \otimes \quad \otimes \end{array}$	$\wedge \mathbb{F} 1$

□

fol:tab:ppr:
prop:provability-lor

Proposition tab.2.

1. $\varphi \vee \psi, \neg \varphi, \neg \psi$ is inconsistent.
2. Both $\varphi \vdash \varphi \vee \psi$ and $\psi \vdash \varphi \vee \psi$.

Proof. 1. We give a closed **tableau** of $\{\mathbb{T} \varphi \vee \psi, \mathbb{T} \neg \varphi, \mathbb{T} \neg \psi\}$:

1.	$\mathbb{T} \varphi \vee \psi$	Assumption
2.	$\mathbb{T} \neg \varphi$	Assumption
3.	$\mathbb{T} \neg \psi$	Assumption
4.	$\mathbb{F} \varphi$	$\neg \mathbb{T} 2$
5.	$\mathbb{F} \psi$	$\neg \mathbb{T} 3$
$\swarrow \quad \searrow$		
6.	$\mathbb{T} \varphi \quad \mathbb{T} \psi$	$\vee \mathbb{T} 1$
	$\otimes \quad \otimes$	

2. Both $\{\mathbb{F} \varphi \vee \psi, \mathbb{T} \varphi\}$ and $\{\mathbb{F} \varphi \vee \psi, \mathbb{T} \psi\}$ have closed **tableaux**:

1.	$\mathbb{F} \varphi \vee \psi$	Assumption
2.	$\mathbb{T} \varphi$	Assumption
3.	$\mathbb{F} \varphi$	$\vee \mathbb{F} 1$
4.	$\mathbb{F} \psi$	$\vee \mathbb{F} 1$
	\otimes	

1.	$\mathbb{F} \varphi \vee \psi$	Assumption
2.	$\mathbb{T} \psi$	Assumption
3.	$\mathbb{F} \varphi$	$\vee \mathbb{F} 1$
4.	$\mathbb{F} \psi$	$\vee \mathbb{F} 1$
	\otimes	

□

Proposition tab.3.

1. $\varphi, \varphi \rightarrow \psi \vdash \psi$.
2. Both $\neg \varphi \vdash \varphi \rightarrow \psi$ and $\psi \vdash \varphi \rightarrow \psi$.

Proof. 1. $\{\mathbb{F} \psi, \mathbb{T} \varphi \rightarrow \psi, \mathbb{T} \varphi\}$ has a closed **tableau**:

1.	$\mathbb{F} \psi$	Assumption
2.	$\mathbb{T} \varphi \rightarrow \psi$	Assumption
3.	$\mathbb{T} \varphi$	Assumption
$\swarrow \quad \searrow$		
4.	$\mathbb{F} \varphi \quad \mathbb{T} \psi$	$\rightarrow \mathbb{T} 2$
	$\otimes \quad \otimes$	

2. Both $\{\mathbb{F} \varphi \rightarrow \psi, \mathbb{T} \neg \varphi\}$ and $\{\mathbb{F} \varphi \rightarrow \psi, \mathbb{T} \neg \psi\}$ have closed **tableaux**:

fol:tab:ppr:
 prop:provability-lif
 fol:tab:ppr:
 prop:provability-lif-left
 fol:tab:ppr:
 prop:provability-lif-right

1. $\mathbb{F} \varphi \rightarrow \psi$ Assumption
 2. $\mathbb{T} \neg \varphi$ Assumption
 3. $\mathbb{T} \varphi$ $\rightarrow \mathbb{F} 1$
 4. $\mathbb{F} \psi$ $\rightarrow \mathbb{F} 1$
 5. $\mathbb{F} \varphi$ $\neg \mathbb{T} 2$
- \otimes

1. $\mathbb{F} \varphi \rightarrow \psi$ Assumption
 2. $\mathbb{T} \neg \psi$ Assumption
 3. $\mathbb{T} \varphi$ $\rightarrow \mathbb{F} 1$
 4. $\mathbb{F} \psi$ $\rightarrow \mathbb{F} 1$
 5. $\mathbb{F} \psi$ $\neg \mathbb{T} 2$
- \otimes

□

Photo Credits

Bibliography