

## tab.1 Derivability and the Propositional Connectives

fol:tab:ppr:  
sec

### Proposition tab.1.

fol:tab:ppr:  
prop:provability-land

fol:tab:ppr:  
prop:provability-land-left

fol:tab:ppr:  
prop:provability-land-right

1. Both  $\varphi \wedge \psi \vdash \varphi$  and  $\varphi \wedge \psi \vdash \psi$ .
2.  $\varphi, \psi \vdash \varphi \wedge \psi$ .

*Proof.* 1. Both  $\{\mathbb{F} \varphi, \mathbb{T} \varphi \wedge \psi\}$  and  $\{\mathbb{F} \psi, \mathbb{T} \varphi \wedge \psi\}$  have closed **tableaux**

1.	$\mathbb{F} \varphi$	Assumption
2.	$\mathbb{T} \varphi \wedge \psi$	Assumption
3.	$\mathbb{T} \varphi$	$\wedge \mathbb{T} 2$
4.	$\mathbb{T} \psi$	$\wedge \mathbb{T} 2$
	$\otimes$	

1.	$\mathbb{F} \psi$	Assumption
2.	$\mathbb{T} \varphi \wedge \psi$	Assumption
3.	$\mathbb{T} \varphi$	$\wedge \mathbb{T} 2$
4.	$\mathbb{T} \psi$	$\wedge \mathbb{T} 2$
	$\otimes$	

2. Here is a closed **tableau** for  $\{\mathbb{T} \varphi, \mathbb{T} \psi, \mathbb{F} \varphi \wedge \psi\}$ :

1.	$\mathbb{F} \varphi \wedge \psi$	Assumption
2.	$\mathbb{T} \varphi$	Assumption
3.	$\mathbb{T} \psi$	Assumption
4.	$\begin{array}{c} \diagdown \quad \diagup \\ \mathbb{F} \varphi \quad \mathbb{F} \psi \\ \otimes \quad \otimes \end{array}$	$\wedge \mathbb{F} 1$

□

fol:tab:ppr:  
prop:provability-lor

### Proposition tab.2.

1.  $\varphi \vee \psi, \neg \varphi, \neg \psi$  is inconsistent.
2. Both  $\varphi \vdash \varphi \vee \psi$  and  $\psi \vdash \varphi \vee \psi$ .

*Proof.* 1. We give a closed **tableau** of  $\{\mathbb{T} \varphi \vee \psi, \mathbb{T} \neg \varphi, \mathbb{T} \neg \psi\}$ :

1.	$\mathbb{T} \varphi \vee \psi$	Assumption				
2.	$\mathbb{T} \neg \varphi$	Assumption				
3.	$\mathbb{T} \neg \psi$	Assumption				
4.	$\mathbb{F} \varphi$	$\neg \mathbb{T} 2$				
5.	$\mathbb{F} \psi$	$\neg \mathbb{T} 3$				
$\swarrow \quad \searrow$						
6.	<table style="display: inline-table; vertical-align: middle;"> <tr> <td style="padding-right: 10px;"><math>\mathbb{T} \varphi</math></td> <td style="padding-right: 10px;"><math>\mathbb{T} \psi</math></td> </tr> <tr> <td style="text-align: center;"><math>\otimes</math></td> <td style="text-align: center;"><math>\otimes</math></td> </tr> </table>	$\mathbb{T} \varphi$	$\mathbb{T} \psi$	$\otimes$	$\otimes$	$\vee \mathbb{T} 1$
$\mathbb{T} \varphi$	$\mathbb{T} \psi$					
$\otimes$	$\otimes$					

2. Both  $\{\mathbb{F} \varphi \vee \psi, \mathbb{T} \varphi\}$  and  $\{\mathbb{F} \varphi \vee \psi, \mathbb{T} \psi\}$  have closed **tableaux**:

1.	$\mathbb{F} \varphi \vee \psi$	Assumption
2.	$\mathbb{T} \varphi$	Assumption
3.	$\mathbb{F} \varphi$	$\vee \mathbb{F} 1$
4.	$\mathbb{F} \psi$	$\vee \mathbb{F} 1$
$\otimes$		

1.	$\mathbb{F} \varphi \vee \psi$	Assumption
2.	$\mathbb{T} \psi$	Assumption
3.	$\mathbb{F} \varphi$	$\vee \mathbb{F} 1$
4.	$\mathbb{F} \psi$	$\vee \mathbb{F} 1$
$\otimes$		

□

**Proposition tab.3.**

1.  $\varphi, \varphi \rightarrow \psi \vdash \psi$ .
2. Both  $\neg \varphi \vdash \varphi \rightarrow \psi$  and  $\psi \vdash \varphi \rightarrow \psi$ .

*Proof.* 1.  $\{\mathbb{F} \psi, \mathbb{T} \varphi \rightarrow \psi, \mathbb{T} \varphi\}$  has a closed **tableau**:

1.	$\mathbb{F} \psi$	Assumption				
2.	$\mathbb{T} \varphi \rightarrow \psi$	Assumption				
3.	$\mathbb{T} \varphi$	Assumption				
$\swarrow \quad \searrow$						
4.	<table style="display: inline-table; vertical-align: middle;"> <tr> <td style="padding-right: 10px;"><math>\mathbb{F} \varphi</math></td> <td style="padding-right: 10px;"><math>\mathbb{T} \psi</math></td> </tr> <tr> <td style="text-align: center;"><math>\otimes</math></td> <td style="text-align: center;"><math>\otimes</math></td> </tr> </table>	$\mathbb{F} \varphi$	$\mathbb{T} \psi$	$\otimes$	$\otimes$	$\rightarrow \mathbb{T} 2$
$\mathbb{F} \varphi$	$\mathbb{T} \psi$					
$\otimes$	$\otimes$					

2. Both  $\{\mathbb{F} \varphi \rightarrow \psi, \mathbb{T} \neg \varphi\}$  and  $\{\mathbb{F} \varphi \rightarrow \psi, \mathbb{T} \neg \psi\}$  have closed **tableaux**:

*fol:tab:ppr:*  
*prop:provability-lif*  
*fol:tab:ppr:*  
*prop:provability-lif-left*  
*fol:tab:ppr:*  
*prop:provability-lif-right*

1.  $\mathbb{F} \varphi \rightarrow \psi$  Assumption
  2.  $\mathbb{T} \neg \varphi$  Assumption
  3.  $\mathbb{T} \varphi$   $\rightarrow \mathbb{F} 1$
  4.  $\mathbb{F} \psi$   $\rightarrow \mathbb{F} 1$
  5.  $\mathbb{F} \varphi$   $\neg \mathbb{T} 2$
- $\otimes$

1.  $\mathbb{F} \varphi \rightarrow \psi$  Assumption
  2.  $\mathbb{T} \neg \psi$  Assumption
  3.  $\mathbb{T} \varphi$   $\rightarrow \mathbb{F} 1$
  4.  $\mathbb{F} \psi$   $\rightarrow \mathbb{F} 1$
  5.  $\mathbb{F} \psi$   $\neg \mathbb{T} 2$
- $\otimes$

□

## Photo Credits

## Bibliography