tab.1 Derivability and the Propositional Connectives

fol:tab:ppr: sec

fol:tab:ppr: prop:provability-land

Proposition tab.1.

fol:tab:ppr: prop:provability-land-left

1. *Both* $\varphi \land \psi \vdash \varphi$ *and* $\varphi \land \psi \vdash \psi$.

fol:tab:ppr: prop:provability-land-right

2. $\varphi, \psi \vdash \varphi \land \psi$.

Proof. 1. Both $\{\mathbb{F}\,\varphi, \mathbb{T}\,\varphi \land \psi\}$ and $\{\mathbb{F}\,\psi, \mathbb{T}\,\varphi \land \psi\}$ have closed tableaux

$$\begin{array}{lcl}
1. & \mathbb{F}\,\varphi & \text{Assumption} \\
2. & \mathbb{T}\,\varphi \land \psi & \text{Assumption} \\
3. & \mathbb{T}\varphi & \land\mathbb{T}\,2 \\
4. & \mathbb{T}\psi & \land\mathbb{T}\,2 \\
 & \otimes &
\end{array}$$

$$\begin{array}{lcl}
1. & \mathbb{F}\,\psi & \text{Assumption} \\
2. & \mathbb{T}\,\varphi \land \psi & \text{Assumption} \\
3. & \mathbb{T}\varphi & \land\mathbb{T}\,2 \\
4. & \mathbb{T}\psi & \land\mathbb{T}\,2 \\
 & \otimes &
\end{array}$$

2. Here is a closed tableau for $\{\mathbb{T}\,\varphi, \mathbb{T}\,\psi, \mathbb{F}\,\varphi \land \psi\}$:

$$\begin{array}{lccl}
1. & \multicolumn{2}{c}{\mathbb{F}\,\varphi \land \psi} & \text{Assumption} \\
2. & \multicolumn{2}{c}{\mathbb{T}\,\varphi} & \text{Assumption} \\
3. & \multicolumn{2}{c}{\mathbb{T}\,\psi} & \text{Assumption} \\
4. & \mathbb{F}\,\varphi & \mathbb{F}\,\psi & \land\mathbb{F}\,1 \\
 & \otimes & \otimes &
\end{array}$$

□

fol:tab:ppr: prop:provability-lor

Proposition tab.2.

1. $\varphi \lor \psi, \neg\varphi, \neg\psi$ *is inconsistent.*

2. *Both* $\varphi \vdash \varphi \lor \psi$ *and* $\psi \vdash \varphi \lor \psi$.

Proof. 1. We give a closed tableau of $\{\mathbb{T}\,\varphi \lor \psi, \mathbb{T}\,\neg\varphi, \mathbb{T}\,\neg\psi\}$:

$$
\begin{array}{rccl}
1. & \multicolumn{2}{c}{\mathbb{T}\,\varphi \lor \psi} & \text{Assumption}\\
2. & \multicolumn{2}{c}{\mathbb{T}\,\neg\varphi} & \text{Assumption}\\
3. & \multicolumn{2}{c}{\mathbb{T}\,\neg\psi} & \text{Assumption}\\
4. & \multicolumn{2}{c}{\mathbb{F}\,\varphi} & \neg\mathbb{T}\,2\\
5. & \multicolumn{2}{c}{\mathbb{F}\,\psi} & \neg\mathbb{T}\,3\\
6. & \mathbb{T}\,\varphi & \mathbb{T}\,\psi & \lor\mathbb{T}\,1\\
 & \otimes & \otimes &
\end{array}
$$

2. Both $\{\mathbb{F}\,\varphi \lor \psi, \mathbb{T}\,\varphi\}$ and $\{\mathbb{F}\,\varphi \lor \psi, \mathbb{T}\,\psi\}$ have closed tableaux:

$$
\begin{array}{rcl}
1. & \mathbb{F}\,\varphi \land \psi & \text{Assumption}\\
2. & \mathbb{T}\,\varphi & \text{Assumption}\\
3. & \mathbb{F}\varphi & \lor\mathbb{F}\,2\\
4. & \mathbb{F}\psi & \lor\mathbb{F}\,2\\
 & \otimes &
\end{array}
$$

$$
\begin{array}{rcl}
1. & \mathbb{F}\,\varphi \land \psi & \text{Assumption}\\
2. & \mathbb{T}\,\psi & \text{Assumption}\\
3. & \mathbb{F}\varphi & \lor\mathbb{F}\,2\\
4. & \mathbb{F}\psi & \lor\mathbb{F}\,2\\
 & \otimes &
\end{array}
$$

□

Proposition tab.3. *fol:tab:ppr: prop:provability-lif*

1. $\varphi, \varphi \to \psi \vdash \psi$. *fol:tab:ppr: prop:provability-lif-left*
2. *Both* $\neg\varphi \vdash \varphi \to \psi$ *and* $\psi \vdash \varphi \to \psi$. *fol:tab:ppr: prop:provability-lif-right*

Proof. 1. $\{\mathbb{F}\,\psi, \mathbb{T}\,\varphi \to \psi, \mathbb{T}\,\varphi\}$ has a closed tableau:

$$
\begin{array}{rccl}
1. & \multicolumn{2}{c}{\mathbb{F}\,\psi} & \text{Assumption}\\
2. & \multicolumn{2}{c}{\mathbb{T}\,\varphi \to \psi} & \text{Assumption}\\
3. & \multicolumn{2}{c}{\mathbb{T}\,\varphi} & \text{Assumption}\\
4. & \mathbb{F}\,\varphi & \mathbb{T}\,\psi & \to\mathbb{T}\,2\\
 & \otimes & \otimes &
\end{array}
$$

2. Both s$\{\mathbb{F}\,\varphi \to \psi, \mathbb{T}\,\neg\varphi\}$ and $\{\mathbb{F}\,\varphi \to \psi, \mathbb{T}\,\neg\psi\}$ have closed tableaux:

1.	$\mathbb{F}\, \varphi \to \psi$	Assumption
2.	$\mathbb{T}\, \neg\varphi$	Assumption
3.	$\mathbb{T}\, \varphi$	$\to\mathbb{F}\, 1$
4.	$\mathbb{F}\, \psi$	$\to\mathbb{F}\, 1$
5.	$\mathbb{F}\, \varphi$	$\neg\mathbb{T}\, 2$
	$\otimes$	

1.	$\mathbb{F}\, \varphi \to \psi$	Assumption
2.	$\mathbb{T}\, \neg\psi$	Assumption
3.	$\mathbb{T}\, \varphi$	$\to\mathbb{F}\, 1$
4.	$\mathbb{F}\, \psi$	$\to\mathbb{F}\, 1$
5.	$\mathbb{F}\, \psi$	$\neg\mathbb{T}\, 2$
	$\otimes$	

$\square$

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