

tab.1 Tableaux with Identity predicate

fol:tab:ide: **Tableaux with identity predicate** require additional inference rules. The rules
 sec for = are (t , t_1 , and t_2 are closed terms):

$$\boxed{\begin{array}{ccc} \frac{}{\mathbb{T} t = t} = & \frac{\mathbb{T} t_1 = t_2}{\mathbb{T} \varphi(t_1)} = \mathbb{T} & \frac{\mathbb{T} t_1 = t_2}{\frac{\mathbb{F} \varphi(t_1)}{\mathbb{F} \varphi(t_2)}} = \mathbb{F} \end{array}}$$

Note that in contrast to all the other rules, $=\mathbb{T}$ and $=\mathbb{F}$ require that *two* signed **formulas** already appear on the branch, namely both $\mathbb{T} t_1 = t_2$ and $\mathbb{F} \varphi(t_1)$.

Example tab.1. If s and t are closed terms, then $s = t, \varphi(s) \vdash \varphi(t)$:

1. $\mathbb{F} \varphi(t)$ Assumption
 2. $\mathbb{T} s = t$ Assumption
 3. $\mathbb{T} \varphi(s)$ Assumption
 4. $\mathbb{T} \varphi(t)$ $=\mathbb{T} 2, 3$
- ⊗

This may be familiar as the principle of substitutability of identicals, or Leibniz' Law.

Tableaux prove that = is symmetric, i.e., that $s_1 = s_2 \vdash s_2 = s_1$:

1. $\mathbb{F} s_2 = s_1$ Assumption
 2. $\mathbb{T} s_1 = s_2$ Assumption
 3. $\mathbb{T} s_1 = s_1$ =
 4. $\mathbb{T} s_2 = s_1$ $=\mathbb{T} 2, 3$
- ⊗

Here, line 2 is the first prerequisite **formula** $\mathbb{T} s_1 = s_2$ of $=\mathbb{T}$. Line 3 is the second one, of the form $\mathbb{T} \varphi(s_2)$ —think of $\varphi(x)$ as $x = s_1$, then $\varphi(s_1)$ is $s_1 = s_1$ and $\varphi(s_2)$ is $s_2 = s_1$.

They also prove that = is transitive, i.e., that $s_1 = s_2, s_2 = s_3 \vdash s_1 = s_3$:

1. $\mathbb{F} s_1 = s_3$ Assumption
 2. $\mathbb{T} s_1 = s_2$ Assumption
 3. $\mathbb{T} s_2 = s_3$ Assumption
 4. $\mathbb{T} s_1 = s_3$ $=\mathbb{T} 3, 2$
- ⊗

In this **tableau**, the first prerequisite **formula** of $=\mathbb{T}$ is line 3, $\mathbb{T} s_2 = s_3$ (s_2 plays the role of t_1 , and s_3 the role of t_2). The second prerequisite, of the

form $\mathbb{T} \varphi(s_2)$ is line 2. Here, think of $\varphi(x)$ as $s_1 = x$; that makes $\varphi(s_2)$ into $t_1 = t_2$ (i.e., line 2) and $\varphi(s_3)$ into the formula $s_1 = s_3$ in the conclusion.

Problem tab.1. Give closed tableaux for the following:

1. $\mathbb{F} \forall x \forall y ((x = y \wedge \varphi(x)) \rightarrow \varphi(y))$
2. $\mathbb{F} \exists x (\varphi(x) \wedge \forall y (\varphi(y) \rightarrow y = x)),$
 $\mathbb{T} \exists x \varphi(x) \wedge \forall y \forall z ((\varphi(y) \wedge \varphi(z)) \rightarrow y = z)$

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Bibliography