

## tab.1 Tableaux with Identity predicate

fol:tab:ide: **Tableaux** with **identity predicate** require additional inference rules. The  
 sec rules for = are ( $t$ ,  $t_1$ , and  $t_2$  are closed terms):

$\frac{}{\mathbb{T} t = t} =$	$\frac{\mathbb{T} t_1 = t_2}{\mathbb{T} \varphi(t_1)} =\mathbb{T}$	$\frac{\mathbb{T} t_1 = t_2}{\frac{\mathbb{F} \varphi(t_1)}{\mathbb{F} \varphi(t_1)} =\mathbb{T}} =\mathbb{T}$
-------------------------------	--	--

Note that in contrast to all the other rules,  $=\mathbb{T}$  and  $=\mathbb{F}$  require that *two* signed **formulas** already appear on the branch, namely both  $\mathbb{T} t_1 = t_2$  and  $\mathbb{F} \varphi(t_1)$ .

**Example tab.1.** If  $s$  and  $t$  are closed terms, then  $s = t, \varphi(s) \vdash \varphi(t)$ :

1.  $\mathbb{F} \varphi(t)$  Assumption
  2.  $\mathbb{T} s = t$  Assumption
  3.  $\mathbb{T} \varphi(s)$  Assumption
  4.  $\mathbb{T} \varphi(t)$   $=\mathbb{T} 2, 3$
- ⊗

This may be familiar as the principle of substitutability of identicals, or Leibniz' Law.

**Tableaux** prove that = is symmetric:

1.  $\mathbb{F} t = s$  Assumption
  2.  $\mathbb{T} s = t$  Assumption
  3.  $\mathbb{T} s = s$  =
  4.  $\mathbb{T} t = s$   $=\mathbb{T} 2, 3$
- ⊗

Here, line 2 is the first prerequisite **formula**  $\mathbb{T} s = t$  of  $=\mathbb{T}$ , and line 3 the second one,  $\mathbb{T} \varphi(s)$ —think of  $\varphi(x)$  as  $x = s$ , then  $\varphi(s)$  is  $s = s$  and  $\varphi(t)$  is  $t = s$ .

They also prove that = is transitive:

1.  $\mathbb{F} t_1 = t_3$  Assumption
  2.  $\mathbb{T} t_1 = t_2$  Assumption
  3.  $\mathbb{T} t_2 = t_3$  Assumption
  4.  $\mathbb{T} t_1 = t_3$   $=\mathbb{T} 3, 2$
- ⊗

In this **tableau**, the first prerequisite **formula** of  $=\mathbb{T}$  is line 3,  $\mathbb{T} t_2 = t_3$ . The second one,  $\mathbb{T} \varphi(t_2)$  is line 2. Think of  $\varphi(x)$  as  $t_1 = x$ ; that makes  $\varphi(t_2)$  into  $t_1 = t_2$  and  $\varphi(t_3)$  into  $t_1 = t_3$ .

**Problem tab.1.** Give closed **tableaux** for the following:

1.  $\mathbb{F} \forall x \forall y ((x = y \wedge \varphi(x)) \rightarrow \varphi(y))$
2.  $\mathbb{F} \exists x (\varphi(x) \wedge \forall y (\varphi(y) \rightarrow y = x)),$   
 $\mathbb{T} \exists x \varphi(x) \wedge \forall y \forall z ((\varphi(y) \wedge \varphi(z)) \rightarrow y = z)$

**Photo Credits**

**Bibliography**