Tableaux with Identity predicate require additional inference rules. The rules for = are (\(t, t_1,\) and \(t_2\) are closed terms):

\[
\begin{align*}
\frac{}{T t = t} & \quad T t_1 = t_2 \\
\frac{T \varphi(t_1)}{} & \quad T \varphi(t_1) = T \\
\frac{T \varphi(t_1)}{F \varphi(t_1)} & \quad F \varphi(t_1) = T
\end{align*}
\]

Note that in contrast to all the other rules, \(=\) and \(\neq\) require that two signed formulas already appear on the branch, namely both \(T t_1 = t_2\) and \(S \varphi(t_1)\).

**Example tab.1.** If \(s\) and \(t\) are closed terms, then \(s = t, \varphi(s) \vdash \varphi(t)\):

1. \(F \varphi(t)\) Assumption
2. \(T s = t\) Assumption
3. \(T \varphi(s)\) Assumption
4. \(T \varphi(t)\) \(\vdash T \ 2, 3\)

This may be familiar as the principle of substitutability of identicals, or Leibniz’ Law.

Tableaux prove that \(=\) is symmetric:

1. \(F t = s\) Assumption
2. \(T s = t\) Assumption
3. \(T s = s\) \(\vdash\)
4. \(T t = s\) \(\vdash T \ 2, 3\)

Here, line 2 is the first prerequisite formula \(T s = t\) of \(=\), and line 3 the second one, \(T \varphi(s)\)—think of \(\varphi(x)\) as \(x = s\), then \(\varphi(s)\) is \(s = s\) and \(\varphi(t)\) is \(t = s\).

They also prove that \(=\) is transitive:

1. \(F t_1 = t_3\) Assumption
2. \(T t_1 = t_2\) Assumption
3. \(T t_2 = t_3\) Assumption
4. \(T t_1 = t_3\) \(\vdash T \ 3, 2\)

In this tableau, the first prerequisite formula of \(=\) is line 3, \(T t_2 = t_3\). The second one, \(T \varphi(t_2)\) is line 2. Think of \(\varphi(x)\) as \(t_1 = x\); that makes \(\varphi(t_2)\) into \(t_1 = t_2\) and \(\varphi(t_3)\) into \(t_1 = t_3\).
Problem tab.1. Give closed tableaux for the following:

1. $\forall x \forall y ((x = y \land \varphi(x)) \rightarrow \varphi(y))$

2. $\exists x (\varphi(x) \land \forall y (\varphi(y) \rightarrow y = x))$
   $\top \exists x \varphi(x) \land \forall y \forall z ((\varphi(y) \land \varphi(z)) \rightarrow y = z)$

Photo Credits

Bibliography