Tableaux with identity predicate require additional inference rules. The rules for = are (t, t₁, and t₂ are closed terms):

\[
\begin{align*}
Tt & = t \\
Tφ(t₁) & = Tφ(t₂) = T
\end{align*}
\]

Note that in contrast to all the other rules, =T and =F require that two signed formulas already appear on the branch, namely both Tt₁ = t₂ and Sφ(t₁).

**Example tab.1.** If s and t are closed terms, then s = t, φ(s) ⊢ φ(t):

1. Fφ(t) Assumption
2. Ts = t Assumption
3. Tφ(s) Assumption
4. Tφ(t) =T 2, 3

This may be familiar as the principle of substitutability of identicals, or Leibniz’ Law.

Tableaux prove that = is symmetric, i.e., that s₁ = s₂ ⊢ s₂ = s₁:

1. F s₂ = s₁ Assumption
2. Ts₁ = s₂ Assumption
3. Ts₁ = s₁ =
4. Ts₂ = s₁ =T 2, 3

Here, line 2 is the first prerequisite formula Ts₁ = s₂ of =T. Line 3 is the second one, of the form Tφ(s₂)—think of φ(x) as x = s₁, then φ(s₁) is s₁ = s₁ and φ(s₂) is s₂ = s₁.

They also prove that = is transitive, i.e., that s₁ = s₂, s₂ = s₃ ⊢ s₁ = s₃:

1. F s₁ = s₃ Assumption
2. Ts₁ = s₂ Assumption
3. Ts₂ = s₃ Assumption
4. Ts₁ = s₃ =T 3, 2

In this tableau, the first prerequisite formula of =T is line 3, Ts₂ = s₃ (s₂ plays the role of t₁, and s₃ the role of t₂). The second prerequisite, of the
form $T \varphi(s_2)$ is line 2. Here, think of $\varphi(x)$ as $s_1 = x$; that makes $\varphi(s_2)$ into $t_1 = t_2$ (i.e., line 2) and $\varphi(s_3)$ into the formula $s_1 = s_3$ in the conclusion.

**Problem tab.1.** Give closed tableaux for the following:

1. $F \forall x \forall y ((x = y \land \varphi(x)) \rightarrow \varphi(y))$
2. $F \exists x (\varphi(x) \land \forall y (\varphi(y) \rightarrow y = x)),$
   $T \exists x \varphi(x) \land \forall y \forall z ((\varphi(y) \land \varphi(z)) \rightarrow y = z)$

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**Bibliography**