Tableaux

We’ve said what an assumption is, and we’ve given the rules of inference. Tableaux are inductively generated from these: each tableau either is a single branch consisting of one or more assumptions, or it results from a tableau by applying one of the rules of inference on a branch.

Definition (Tableau). A tableau for assumptions $S_{\phi_1}, \ldots, S_{\phi_n}$ (where each $S_i$ is either $T$ or $F$) is a tree of signed formulas satisfying the following conditions:

1. The $n$ topmost signed formulas of the tree are $S_{\phi_1}$, one below the other.
2. Every signed formula in the tree that is not one of the assumptions results from a correct application of an inference rule to a signed formula in the branch above it.

A branch of a tableau is closed iff it contains both $T\phi$ and $F\phi$, and open otherwise. A tableau in which every branch is closed is a closed tableau (for its set of assumptions). If a tableau is not closed, i.e., if it contains at least one open branch, it is open.

Example. Every set of assumptions on its own is a tableau, but it will generally not be closed. (Obviously, it is closed only if the assumptions already contain a pair of signed formulas $T\phi$ and $F\phi$.)

From a tableau (open or closed) we can obtain a new, larger one by applying one of the rules of inference to a signed formula $\phi$ in it. The rule will append one or more signed formulas to the end of any branch containing the occurrence of $\phi$ to which we apply the rule.

For instance, consider the assumption $T\phi \land \lnot \phi$. Here is the (open) tableau consisting of just that assumption:

1. $T\phi \land \lnot \phi$ Assumption

We obtain a new tableau from it by applying the $\land T$ rule to the assumption. That rule allows us to add two new lines to the tableau, $T\phi$ and $T\lnot \phi$:

1. $T\phi \land \lnot \phi$ Assumption
2. $T\phi$ $\land T$ 1
3. $T\lnot \phi$ $\land T$ 1

When we write down tableaux, we record the rules we’ve applied on the right (e.g., $\land T$ 1 means that the signed formula on that line is the result of applying the $\land T$ rule to the signed formula on line 1). This new tableau now contains additional signed formulas, but to only one ($T\lnot \phi$) can we apply a rule (in this case, the $\lnot T$ rule). This results in the closed tableau.
1. $T \varphi \land \neg \varphi$  Assumption
2. $T \varphi$  $\land T 1$
3. $T \neg \varphi$  $\land T 1$
4. $F \varphi$  $\neg T 3$

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Bibliography