

tab.1 Tableaux

fol:tab:der: We've said what an assumption is, and we've given the rules of inference. explanation
sec

Tableaux are inductively generated from these: each **tableau** either is a single branch consisting of one or more assumptions, or it results from a **tableau** by applying one of the rules of inference on a branch.

Definition tab.1 (Tableau). A **tableau** for assumptions $S_1\varphi_1, \dots, S_n\varphi_n$ (where each S_i is either \mathbb{T} or \mathbb{F}) is a tree of **signed formulas** satisfying the following conditions:

1. The n topmost **signed formulas** of the tree are $S_i\varphi_i$, one below the other.
2. Every **signed formula** in the tree that is not one of the assumptions results from a correct application of an inference rule to a **signed formula** in the branch above it.

A branch of a **tableau** is *closed* iff it contains both $\mathbb{T}\varphi$ and $\mathbb{F}\varphi$, and *open* otherwise. A **tableau** in which every branch is closed is a *closed tableau* (for its set of assumptions). If a **tableau** is not closed, i.e., if it contains at least one open branch, it is *open*.

Example tab.2. Every set of assumptions on its own is a **tableau**, but it will generally not be closed. (Obviously, it is closed only if the assumptions already contain a pair of **signed formulas** $\mathbb{T}\varphi$ and $\mathbb{F}\varphi$.)

From a **tableau** (open or closed) we can obtain a new, larger one by applying one of the rules of inference to a **signed formula** φ in it. The rule will append one or more **signed formulas** to the end of any branch containing the occurrence of φ to which we apply the rule.

For instance, consider the assumption $\mathbb{T}\varphi \wedge \neg\varphi$. Here is the (open) **tableau** consisting of just that assumption:

1. $\mathbb{T}\varphi \wedge \neg\varphi$ Assumption

We obtain a new **tableau** from it by applying the $\wedge\mathbb{T}$ rule to the assumption. That rule allows us to add two new lines to the **tableau**, $\mathbb{T}\varphi$ and $\mathbb{T}\neg\varphi$:

1. $\mathbb{T}\varphi \wedge \neg\varphi$ Assumption
2. $\mathbb{T}\varphi$ $\wedge\mathbb{T}1$
3. $\mathbb{T}\neg\varphi$ $\wedge\mathbb{T}1$

When we write down **tableaux**, we record the rules we've applied on the right (e.g., $\wedge\mathbb{T}1$ means that the **signed formula** on that line is the result of applying the $\wedge\mathbb{T}$ rule to the **signed formula** on line 1). This new **tableau** now contains additional **signed formulas**, but to only one ($\mathbb{T}\neg\varphi$) can we apply a rule (in this case, the $\neg\mathbb{T}$ rule). This results in the closed **tableau**

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|----|----------------------------------------|---------------------|
| 1. | $\mathbb{T}\varphi \wedge \neg\varphi$ | Assumption |
| 2. | $\mathbb{T}\varphi$ | $\wedge\mathbb{T}1$ |
| 3. | $\mathbb{T}\neg\varphi$ | $\wedge\mathbb{T}1$ |
| 4. | $\mathbb{F}\varphi$ | $\neg\mathbb{T}3$ |
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Bibliography