tab.1 Tableaux

fol:tab:der:sec We've said what an assumption is, and we've given the rules of inference. Tableaux are inductively generated from these: each tableau either is a single branch consisting of one or more assumptions, or it results from a tableau by applying one of the rules of inference on a branch. explanation

Definition tab.1 (Tableau). A tableau for assumptions $S_1\varphi_1, \dots, S_n\varphi_n$ (where each S_i is either $\mathbb{T}$ or $\mathbb{F}$) is a finite tree of signed formulas satisfying the following conditions:

1. The n topmost signed formulas of the tree are $S_i\varphi_i$, one below the other.
2. Every signed formula in the tree that is not one of the assumptions results from a correct application of an inference rule to a signed formula in the branch above it.

A branch of a tableau is *closed* iff it contains both $\mathbb{T}\,\varphi$ and $\mathbb{F}\,\varphi$, and *open* otherwise. A tableau in which every branch is closed is a *closed tableau* (for its set of assumptions). If a tableau is not closed, i.e., if it contains at least one open branch, it is *open*.

Example tab.2. Every set of assumptions on its own is a tableau, but it will generally not be closed. (Obviously, it is closed only if the assumptions already contain a pair of signed formulas $\mathbb{T}\,\varphi$ and $\mathbb{F}\,\varphi$.)

From a tableau (open or closed) we can obtain a new, larger one by applying one of the rules of inference to a signed formula φ in it. The rule will append one or more signed formulas to the end of any branch containing the occurrence of φ to which we apply the rule.

For instance, consider the assumption $\mathbb{T}\,\varphi \wedge \neg\varphi$. Here is the (open) tableau consisting of just that assumption:

1.	$\mathbb{T}\,\varphi \wedge \neg\varphi$	Assumption

We obtain a new tableau from it by applying the $\wedge\mathbb{T}$ rule to the assumption. That rule allows us to add two new lines to the tableau, $\mathbb{T}\,\varphi$ and $\mathbb{T}\,\neg\varphi$:

1.	$\mathbb{T}\,\varphi \wedge \neg\varphi$	Assumption
2.	$\mathbb{T}\,\varphi$	$\wedge\mathbb{T}\,1$
3.	$\mathbb{T}\,\neg\varphi$	$\wedge\mathbb{T}\,1$

When we write down tableaux, we record the rules we've applied on the right (e.g., $\wedge\mathbb{T}1$ means that the signed formula on that line is the result of applying the $\wedge\mathbb{T}$ rule to the signed formula on line 1). This new tableau now contains additional signed formulas, but to only one ($\mathbb{T}\,\neg\varphi$) can we apply a rule (in this case, the $\neg\mathbb{T}$ rule). This results in the closed tableau

1.	$\mathbb{T}\,\varphi \wedge \neg\varphi$	Assumption
2.	$\mathbb{T}\,\varphi$	$\wedge\mathbb{T}\,1$
3.	$\mathbb{T}\,\neg\varphi$	$\wedge\mathbb{T}\,1$
4.	$\mathbb{F}\,\varphi$	$\neg\mathbb{T}\,3$
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Bibliography