

## syn.1 Main operator of a Formula

fol:syn:mai: It is often useful to talk about the last operator used in constructing a explanation  
 sec formula  $\varphi$ . This operator is called the *main operator* of  $\varphi$ . Intuitively, it is the “outermost” operator of  $\varphi$ . For example, the main operator of  $\neg\varphi$  is  $\neg$ , the main operator of  $(\varphi \vee \psi)$  is  $\vee$ , etc.

fol:syn:mai: **Definition syn.1 (Main operator).** The *main operator* of a formula  $\varphi$  is  
 def:main-op defined as follows:

1.  $\varphi$  is atomic:  $\varphi$  has no main operator.
2.  $\varphi \equiv \neg\psi$ : the main operator of  $\varphi$  is  $\neg$ .
3.  $\varphi \equiv (\psi \wedge \chi)$ : the main operator of  $\varphi$  is  $\wedge$ .
4.  $\varphi \equiv (\psi \vee \chi)$ : the main operator of  $\varphi$  is  $\vee$ .
5.  $\varphi \equiv (\psi \rightarrow \chi)$ : the main operator of  $\varphi$  is  $\rightarrow$ .
6.  $\varphi \equiv (\psi \leftrightarrow \chi)$ : the main operator of  $\varphi$  is  $\leftrightarrow$ .
7.  $\varphi \equiv \forall x \psi$ : the main operator of  $\varphi$  is  $\forall$ .
8.  $\varphi \equiv \exists x \psi$ : the main operator of  $\varphi$  is  $\exists$ .

In each case, we intend the specific indicated *occurrence* of the main operator in the formula. For instance, since the formula  $((\theta \rightarrow \alpha) \rightarrow (\alpha \rightarrow \theta))$  is of the form  $(\psi \rightarrow \chi)$  where  $\psi$  is  $(\theta \rightarrow \alpha)$  and  $\chi$  is  $(\alpha \rightarrow \theta)$ , the second occurrence of  $\rightarrow$  is the main operator.

This is a *recursive* definition of a function which maps all non-atomic formulas to their main operator occurrence. Because of the way formulas are defined inductively, every formula  $\varphi$  satisfies one of the cases in Definition syn.1. This guarantees that for each non-atomic formula  $\varphi$  a main operator exists. Because each formula satisfies only one of these conditions, and because the smaller formulas from which  $\varphi$  is constructed are uniquely determined in each case, the main operator occurrence of  $\varphi$  is unique, and so we have defined a function. explanation

We call formulas by the following names depending on which symbol their main operator is:

Main operator	Type of formula	Example
none	atomic (formula)	$\perp, \top, R(t_1, \dots, t_n), t_1 = t_2$
$\neg$	negation	$\neg\varphi$
$\wedge$	conjunction	$(\varphi \wedge \psi)$
$\vee$	disjunction	$(\varphi \vee \psi)$
$\rightarrow$	conditional	$(\varphi \rightarrow \psi)$
$\forall$	universal (formula)	$\forall x \varphi$
$\exists$	existential (formula)	$\exists x \varphi$

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**Bibliography**