seq.1 Rules and Derivations

For the following, let $\Gamma, \Delta, \Pi, \Lambda$ represent finite sequences of sentences.

**Definition seq.1 (Sequent).** A **sequent** is an expression of the form

$$\Gamma \Rightarrow \Delta$$

where $\Gamma$ and $\Delta$ are finite (possibly empty) sequences of sentences of the language $L$. $\Gamma$ is called the **antecedent**, while $\Delta$ is the **succedent**.

The intuitive idea behind a sequent is: if all of the sentences in the antecedent hold, then at least one of the sentences in the succedent holds. That is, if $\Gamma = \langle \varphi_1, \ldots, \varphi_m \rangle$ and $\Delta = \langle \psi_1, \ldots, \psi_n \rangle$, then $\Gamma \Rightarrow \Delta$ holds iff

$$(\varphi_1 \land \cdots \land \varphi_m) \rightarrow (\psi_1 \lor \cdots \lor \psi_n)$$

holds. There are two special cases: when $\Gamma$ is empty and when $\Delta$ is empty. When $\Gamma$ is empty, i.e., $m = 0$, $\Rightarrow \Delta$ holds iff $\psi_1 \lor \cdots \lor \psi_n$ holds. When $\Delta$ is empty, i.e., $n = 0$, $\Gamma \Rightarrow$ holds iff $\neg (\varphi_1 \land \cdots \land \varphi_m)$ does. We say a sequent is valid iff the corresponding sentence is valid.

If $\Gamma$ is a sequence of sentences, we write $\Gamma, \varphi$ for the result of appending $\varphi$ to the right end of $\Gamma$ (and $\varphi, \Gamma$ for the result of appending $\varphi$ to the left end of $\Gamma$). If $\Delta$ is a sequence of sentences also, then $\Gamma, \Delta$ is the concatenation of the two sequences.

**Definition seq.2 (Initial Sequent).** An **initial sequent** is a sequent of one of the following forms:

1. $\varphi \Rightarrow \varphi$
2. $\Rightarrow \top$
3. $\bot \Rightarrow$

for any sentence $\varphi$ in the language.

**Derivations** in the sequent calculus are certain trees of sequents, where the topmost sequents are initial sequents, and if a sequent stands below one or two other sequents, it must follow correctly by a rule of inference. The rules for LK are divided into two main types: **logical** rules and **structural** rules. The logical rules are named for the main operator of the sentence containing $\varphi$ and/or $\psi$ in the lower sequent. Each one comes in two versions, one for inferring a sequent with the sentence containing the logical operator on the left, and one with the sentence on the right.

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Bibliography