

## seq.1 Rules and Derivations

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sec For the following, let  $\Gamma, \Delta, \Pi, \Lambda$  represent finite sequences of **sentences**.

**Definition seq.1 (Sequent).** A *sequent* is an expression of the form

$$\Gamma \Rightarrow \Delta$$

where  $\Gamma$  and  $\Delta$  are finite (possibly empty) sequences of **sentences** of the language  $\mathcal{L}$ .  $\Gamma$  is called the *antecedent*, while  $\Delta$  is the *succedent*.

The intuitive idea behind a sequent is: if all of the **sentences** in the antecedent hold, then at least one of the **sentences** in the succedent holds. That is, if  $\Gamma = \langle \varphi_1, \dots, \varphi_m \rangle$  and  $\Delta = \langle \psi_1, \dots, \psi_n \rangle$ , then  $\Gamma \Rightarrow \Delta$  holds iff explanation

$$(\varphi_1 \wedge \dots \wedge \varphi_m) \rightarrow (\psi_1 \vee \dots \vee \psi_n)$$

holds. There are two special cases: where  $\Gamma$  is empty and when  $\Delta$  is empty. When  $\Gamma$  is empty, i.e.,  $m = 0$ ,  $\Rightarrow \Delta$  holds iff  $\psi_1 \vee \dots \vee \psi_n$  holds. When  $\Delta$  is empty, i.e.,  $n = 0$ ,  $\Gamma \Rightarrow$  holds iff  $\neg(\varphi_1 \wedge \dots \wedge \varphi_m)$  does. We say a sequent is valid iff the corresponding **sentence** is valid.

If  $\Gamma$  is a sequence of **sentences**, we write  $\Gamma, \varphi$  for the result of appending  $\varphi$  to the right end of  $\Gamma$  (and  $\varphi, \Gamma$  for the result of appending  $\varphi$  to the left end of  $\Gamma$ ). If  $\Delta$  is a sequence of **sentences** also, then  $\Gamma, \Delta$  is the concatenation of the two sequences.

**Definition seq.2 (Initial Sequent).** An *initial sequent* is a sequent of one of the following forms:

1.  $\varphi \Rightarrow \varphi$
2.  $\Rightarrow \top$
3.  $\perp \Rightarrow$

for any **sentence**  $\varphi$  in the language.

**Derivations** in the sequent calculus are certain trees of sequents, where the topmost sequents are initial sequents, and if a sequent stands below one or two other sequents, it must follow correctly by a rule of inference. The rules for **LK** are divided into two main types: *logical* rules and *structural* rules. The logical rules are named for the **main operator** of the **sentence** containing  $\varphi$  and/or  $\psi$  in the lower sequent. Each one comes in two versions, one for inferring a sequent with the **sentence** containing the **logical operator** on the left, and one with the **sentence** on the right.

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## Bibliography