

seq.1 Quantifier Rules

fol:seq:qrl:
sec

Rules for \forall

$$\frac{\varphi(t), \Gamma \Rightarrow \Delta}{\forall x \varphi(x), \Gamma \Rightarrow \Delta} \forall L \qquad \frac{\Gamma \Rightarrow \Delta, \varphi(a)}{\Gamma \Rightarrow \Delta, \forall x \varphi(x)} \forall R$$

In $\forall L$, t is a closed term (i.e., one without variables). In $\forall R$, a is a **constant symbol** which must not occur anywhere in the lower sequent of the $\forall R$ rule. We call a the *eigenvariable* of the $\forall R$ inference.

Rules for \exists

$$\frac{\varphi(a), \Gamma \Rightarrow \Delta}{\exists x \varphi(x), \Gamma \Rightarrow \Delta} \exists L \qquad \frac{\Gamma \Rightarrow \Delta, \varphi(t)}{\Gamma \Rightarrow \Delta, \exists x \varphi(x)} \exists R$$

Again, t is a closed term, and a is a **constant symbol** which does not occur in the lower sequent of the $\exists L$ rule. We call a the *eigenvariable* of the $\exists L$ inference.

The condition that an eigenvariable not occur in the lower sequent of the $\forall R$ or $\exists L$ inference is called the *eigenvariable condition*.

We use the term “eigenvariable” even though a in the above rules is a **constant symbol**. This has historical reasons. explanation

In $\exists R$ and $\forall L$ there are no restrictions on the term t . On the other hand, in the $\exists L$ and $\forall R$ rules, the eigenvariable condition requires that the **constant symbol** a does not occur anywhere outside of $\varphi(a)$ in the upper sequent. It is necessary to ensure that the system is sound, i.e., only **derives** sequents that are valid. Without this condition, the following would be allowed:

$$\frac{\varphi(a) \Rightarrow \varphi(a)}{\exists x \varphi(x) \Rightarrow \varphi(a)} * \exists L \qquad \frac{\varphi(a) \Rightarrow \varphi(a)}{\varphi(a) \Rightarrow \forall x \varphi(x)} * \forall R$$
$$\frac{\exists x \varphi(x) \Rightarrow \varphi(a)}{\exists x \varphi(x) \Rightarrow \forall x \varphi(x)} \forall R \qquad \frac{\varphi(a) \Rightarrow \forall x \varphi(x)}{\exists x \varphi(x) \Rightarrow \forall x \varphi(x)} \exists L$$

However, $\exists x \varphi(x) \Rightarrow \forall x \varphi(x)$ is not valid.

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Bibliography