seq.1 Derivability and the Quantifiers

fol:seq:qpr:sec The completeness theorem also requires that the sequent calculus rules rules yield the facts about $\vdash$ established in this section. explanation

fol:seq:qpr:thm:strong-generalization **Theorem seq.1.** *If c is a constant not occurring in Γ or $\varphi(x)$ and $\Gamma \vdash \varphi(c)$, then $\Gamma \vdash \forall x\, \varphi(x)$.*

Proof. Let π_0 be an **LK**-derivation of $\Gamma_0 \Rightarrow \varphi(c)$ for some finite $\Gamma_0 \subseteq \Gamma$. By adding a $\forall$R inference, we obtain a derivation of $\Gamma_0 \Rightarrow \forall x\, \varphi(x)$, since c does not occur in Γ or $\varphi(x)$ and thus the eigenvariable condition is satisfied. □

fol:seq:qpr:prop:provability-quantifiers **Proposition seq.2.**

1. $\varphi(t) \vdash \exists x\, \varphi(x)$.
2. $\forall x\, \varphi(x) \vdash \varphi(t)$.

Proof.
1. The sequent $\varphi(t) \Rightarrow \exists x\, \varphi(x)$ is derivable:

$$\dfrac{\varphi(t) \Rightarrow \varphi(t)}{\varphi(t) \Rightarrow \exists x\, \varphi(x)}\ \exists\mathrm{R}$$

2. The sequent $\forall x\, \varphi(x) \Rightarrow \varphi(t)$ is derivable:

$$\dfrac{\varphi(t) \Rightarrow \varphi(t)}{\forall x\, \varphi(x) \Rightarrow \varphi(t)}\ \forall\mathrm{L}$$

□

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Bibliography