

seq.1 Derivability and the Quantifiers

fol:seq:qpr:
sec

fol:seq:qpr:
thm:strong-generalization

Theorem seq.1. *If c is a constant not occurring in Γ or $\varphi(x)$ and $\Gamma \vdash \varphi(c)$, then $\Gamma \vdash \forall x \varphi(x)$.*

Proof. Let π_0 be an **LK-derivation** of $\Gamma_0 \Rightarrow \varphi(c)$ for some finite $\Gamma_0 \subseteq \Gamma$. By adding a $\forall R$ inference, we obtain a proof of $\Gamma_0 \Rightarrow \forall x \varphi(x)$, since c does not occur in Γ or $\varphi(x)$ and thus the eigenvariable condition is satisfied. \square

fol:seq:qpr:
prop:provability-quantifiers

Proposition seq.2.

1. $\varphi(t) \vdash \exists x \varphi(x)$.
2. $\forall x \varphi(x) \vdash \varphi(t)$.

Proof. 1. The sequent $\varphi(t) \Rightarrow \exists x \varphi(x)$ is **derivable**:

$$\frac{\varphi(t) \Rightarrow \varphi(t)}{\varphi(t) \Rightarrow \exists x \varphi(x)} \exists R$$

2. The sequent $\forall x \varphi(x) \Rightarrow \varphi(t)$ is **derivable**:

$$\frac{\varphi(t) \Rightarrow \varphi(t)}{\forall x \varphi(x) \Rightarrow \varphi(t)} \forall L$$

\square

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Bibliography