

## seq.1 Derivability and the Quantifiers

fol:seq:qpr:  
sec

fol:seq:qpr:  
thm:strong-generalization

**Theorem seq.1.** *If  $c$  is a constant not occurring in  $\Gamma$  or  $\varphi(x)$  and  $\Gamma \vdash \varphi(c)$ , then  $\Gamma \vdash \forall x \varphi(x)$ .*

*Proof.* Let  $\pi_0$  be an **LK-derivation** of  $\Gamma_0 \Rightarrow \varphi(c)$  for some finite  $\Gamma_0 \subseteq \Gamma$ . By adding a  $\forall R$  inference, we obtain a proof of  $\Gamma_0 \Rightarrow \forall x \varphi(x)$ , since  $c$  does not occur in  $\Gamma$  or  $\varphi(x)$  and thus the eigenvariable condition is satisfied.  $\square$

fol:seq:qpr:  
prop:provability-quantifiers

**Theorem seq.2.**

1.  $\varphi(t) \vdash \exists x \varphi(x)$ .
2.  $\forall x \varphi(x) \vdash \varphi(t)$ .

*Proof.* 1. The sequent  $\varphi(t) \Rightarrow \exists x \varphi(x)$  is **derivable**:

$$\frac{\varphi(t) \Rightarrow \varphi(t)}{\varphi(t) \Rightarrow \exists x \varphi(x)} \exists R$$

2. The sequent  $\forall x \varphi(x) \Rightarrow \varphi(t)$  is **derivable**:

$$\frac{\varphi(t) \Rightarrow \varphi(t)}{\forall x \varphi(x) \Rightarrow \varphi(t)} \forall L$$

$\square$

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### Bibliography