

seq.1 Derivability and the Propositional Connectives

fol:seq:ppr:
sec

Proposition seq.1.

fol:seq:ppr:

prop:provability-land

fol:seq:ppr:

prop:provability-land-left

fol:seq:ppr:

prop:provability-land-right

1. Both $\varphi \wedge \psi \vdash \varphi$ and $\varphi \wedge \psi \vdash \psi$.

2. $\varphi, \psi \vdash \varphi \wedge \psi$.

Proof. 1. Both sequents $\varphi \wedge \psi \Rightarrow \varphi$ and $\varphi \wedge \psi \Rightarrow \psi$ are **derivable**:

$$\frac{\varphi \Rightarrow \varphi}{\varphi \wedge \psi \Rightarrow \varphi} \wedge L \quad \frac{\psi \Rightarrow \psi}{\varphi \wedge \psi \Rightarrow \psi} \wedge L$$

2. Here is a **derivation** of the sequent $\varphi, \psi \Rightarrow \varphi \wedge \psi$:

$$\frac{\varphi \Rightarrow \varphi \quad \psi \Rightarrow \psi}{\varphi, \psi \Rightarrow \varphi \wedge \psi} \wedge R$$

□

fol:seq:ppr:

prop:provability-lor

Proposition seq.2.

1. $\varphi \vee \psi, \neg\varphi, \neg\psi$ is inconsistent.

2. Both $\varphi \vdash \varphi \vee \psi$ and $\psi \vdash \varphi \vee \psi$.

Proof. 1. We give a **derivation** of the sequent $\varphi \vee \psi, \neg\varphi, \neg\psi \Rightarrow$:

$$\frac{\frac{\frac{\varphi \Rightarrow \varphi}{\neg\varphi, \varphi \Rightarrow} \neg L}{\varphi, \neg\varphi, \neg\psi \Rightarrow} \quad \frac{\frac{\frac{\psi \Rightarrow \psi}{\neg\psi, \psi \Rightarrow} \neg L}{\psi, \neg\varphi, \neg\psi \Rightarrow} \quad \frac{\varphi \vee \psi, \neg\varphi, \neg\psi \Rightarrow} \vee L$$

(Recall that double inference lines indicate several weakening, contraction, and exchange inferences.)

2. Both sequents $\varphi \Rightarrow \varphi \vee \psi$ and $\psi \Rightarrow \varphi \vee \psi$ have **derivations**:

$$\frac{\varphi \Rightarrow \varphi}{\varphi \Rightarrow \varphi \vee \psi} \vee R \quad \frac{\psi \Rightarrow \psi}{\psi \Rightarrow \varphi \vee \psi} \vee R$$

□

fol:seq:ppr:

prop:provability-lif

Proposition seq.3.

fol:seq:ppr:

prop:provability-lif-left

fol:seq:ppr:

prop:provability-lif-right

1. $\varphi, \varphi \rightarrow \psi \vdash \psi$.

2. Both $\neg\varphi \vdash \varphi \rightarrow \psi$ and $\psi \vdash \varphi \rightarrow \psi$.

Proof. 1. The sequent $\varphi \rightarrow \psi, \varphi \Rightarrow \psi$ is **derivable**:

$$\frac{\varphi \Rightarrow \varphi \quad \psi \Rightarrow \psi}{\varphi \rightarrow \psi, \varphi \Rightarrow \psi} \rightarrow\text{L}$$

2. Both sequents $\neg\varphi \Rightarrow \varphi \rightarrow \psi$ and $\psi \Rightarrow \varphi \rightarrow \psi$ are **derivable**:

$$\frac{\frac{\frac{\varphi \Rightarrow \varphi}{\neg\varphi, \varphi \Rightarrow} \neg\text{L}}{\varphi, \neg\varphi \Rightarrow} \text{XL}}{\varphi, \neg\varphi \Rightarrow \psi} \text{WR} \quad \frac{\psi \Rightarrow \psi}{\varphi, \psi \Rightarrow \psi} \text{WL} \rightarrow\text{R}$$

$$\frac{\varphi, \neg\varphi \Rightarrow \psi}{\neg\varphi \Rightarrow \varphi \rightarrow \psi} \rightarrow\text{R} \quad \frac{\psi \Rightarrow \varphi \rightarrow \psi}{\psi \Rightarrow \varphi \rightarrow \psi} \rightarrow\text{R}$$

□

Photo Credits

Bibliography