

## seq.1 Derivations with Identity predicate

fol:seq:ide: Derivations with identity predicate require additional initial sequents and  
sec inference rules.

**Definition seq.1** (Initial sequents for =). If  $t$  is a closed term, then  $\Rightarrow t = t$  is an initial sequent.

The rules for = are ( $t_1$  and  $t_2$  are closed terms):

$$\frac{t_1 = t_2, \Gamma \Rightarrow \Delta, \varphi(t_1)}{t_1 = t_2, \Gamma \Rightarrow \Delta, \varphi(t_2)} = \qquad \frac{t_1 = t_2, \Gamma \Rightarrow \Delta, \varphi(t_2)}{t_1 = t_2, \Gamma \Rightarrow \Delta, \varphi(t_1)} =$$

**Example seq.2.** If  $s$  and  $t$  are closed terms, then  $s = t, \varphi(s) \vdash \varphi(t)$ :

$$\frac{\frac{\varphi(s) \Rightarrow \varphi(s)}{s = t, \varphi(s) \Rightarrow \varphi(s)} \text{WL}}{s = t, \varphi(s) \Rightarrow \varphi(t)} =$$

This may be familiar as the principle of substitutability of identicals, or Leibniz' Law.

**LK** proves that = is symmetric and transitive:

$$\frac{\frac{\Rightarrow t_1 = t_1}{t_1 = t_2 \Rightarrow t_1 = t_1} \text{WL}}{t_1 = t_2 \Rightarrow t_2 = t_1} = \qquad \frac{\frac{t_1 = t_2 \Rightarrow t_1 = t_2}{t_2 = t_3, t_1 = t_2 \Rightarrow t_1 = t_2} \text{WL}}{t_2 = t_3, t_1 = t_2 \Rightarrow t_1 = t_3} = \text{XL}$$

In the proof on the left, the formula  $x = t_1$  is our  $\varphi(x)$ . On the right, we take  $\varphi(x)$  to be  $t_1 = x$ .

**Problem seq.1.** Give derivations of the following sequents:

1.  $\Rightarrow \forall x \forall y ((x = y \wedge \varphi(x)) \rightarrow \varphi(y))$
2.  $\exists x \varphi(x) \wedge \forall y \forall z ((\varphi(y) \wedge \varphi(z)) \rightarrow y = z) \Rightarrow \exists x (\varphi(x) \wedge \forall y (\varphi(y) \rightarrow y = x))$

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## Bibliography