

## prf.1 Tableaux

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sec

While many **derivation** systems operate with arrangements of **sentences**, **tableaux** operate with **signed formulas**. A **signed formula** is a pair consisting of a truth value sign ( $\mathbb{T}$  or  $\mathbb{F}$ ) and a **sentence**

$$\mathbb{T} \varphi \text{ or } \mathbb{F} \varphi.$$

A **tableau** consists of **signed formulas** arranged in a downward-branching tree. It begins with a number of *assumptions* and continues with **signed formulas** which result from one of the **signed formulas** above it by applying one of the rules of inference. Each rule allows us to add one or more **signed formulas** to the end of a branch, or two **signed formulas** side by side—in this case a branch splits into two, with the two added **signed formulas** forming the ends of the two branches.

A rule applied to a complex **signed formula** results in the addition of **signed formulas** which are immediate sub-formulas. They come in pairs, one rule for each of the two signs. For instance, the  $\wedge\mathbb{T}$  rule applies to  $\mathbb{T} \varphi \wedge \psi$ , and allows the addition of both the two **signed formulas**  $\mathbb{T} \varphi$  and  $\mathbb{T} \psi$  to the end of any branch containing  $\mathbb{T} \varphi \wedge \psi$ , and the rule  $\varphi \wedge \psi\mathbb{F}$  allows a branch to be split by adding  $\mathbb{F} \varphi$  and  $\mathbb{F} \psi$  side-by-side. A **tableau** is closed if every one of its branches contains a matching pair of **signed formulas**  $\mathbb{T} \varphi$  and  $\mathbb{F} \varphi$ .

The  $\vdash$  relation based on **tableaux** is defined as follows:  $\Gamma \vdash \varphi$  iff there is some finite set  $\Gamma_0 = \{\psi_1, \dots, \psi_n\} \subseteq \Gamma$  such that there is a closed **tableau** for the assumptions

$$\{\mathbb{F} \varphi, \mathbb{T} \psi_1, \dots, \mathbb{T} \psi_n\}$$

For instance, here is a closed **tableau** that shows that  $\vdash (\varphi \wedge \psi) \rightarrow \varphi$ :

1.	$\mathbb{F} (\varphi \wedge \psi) \rightarrow \varphi$	Assumption
2.	$\mathbb{T} \varphi \wedge \psi$	$\rightarrow\mathbb{F} 1$
3.	$\mathbb{F} \varphi$	$\rightarrow\mathbb{F} 1$
4.	$\mathbb{T} \varphi$	$\rightarrow\mathbb{T} 2$
5.	$\mathbb{T} \psi$	$\rightarrow\mathbb{T} 2$
	$\otimes$	

A set  $\Gamma$  is inconsistent in the **tableau** calculus if there is a closed **tableau** for assumptions

$$\{\mathbb{T} \psi_1, \dots, \mathbb{T} \psi_n\}$$

for some  $\psi_i \in \Gamma$ .

The sequent calculus was invented in the 1950s independently by Evert Beth and Jaakko Hintikka, and simplified and popularized by Raymond Smullyan. It is very easy to use, since constructing a **tableau** is a very systematic procedure. Because of the systematic nature of **tableaux**, they also lend themselves to implementation by computer. However, **tableau** is often hard to read and their connection to proofs are sometimes not easy to see. The approach is also quite

general, and many different logics have **tableau** systems. **Tableaux** also help us to find **structures** that satisfy given (sets of) **sentences**: if the set is satisfiable, it won't have a closed **tableau**, i.e., any **tableau** will have an open branch. The satisfying **structure** can be "read off" an open branch, provided all rules it is possible to apply have been applied on that branch. There is also a very close connection to the sequent calculus: essentially, a closed **tableau** is a condensed **derivation** in the sequent calculus, written upside-down.

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## **Bibliography**