

## ntd.1 Rules and Derivations

fol:ntd:rul:  
sec Natural deduction systems are meant to closely parallel the informal reasoning used in mathematical proof (hence it is somewhat “natural”). explanation Natural deduction proofs begin with assumptions. Inference rules are then applied. Assumptions are “discharged” by the  $\neg$ Intro,  $\rightarrow$ Intro,  $\vee$ Elim and  $\exists$ Elim inference rules, and the label of the discharged assumption is placed beside the inference for clarity.

**Definition ntd.1 (Assumption).** An *assumption* is any sentence in the top-most position of any branch.

Derivations in natural deduction are certain trees of sentences, where the topmost sentences are assumptions, and if a sentence stands below one, two, or three other sequents, it must follow correctly by a rule of inference. The sentences at the top of the inference are called the *premises* and the sentence below the *conclusion* of the inference. The rules come in pairs, an introduction and an elimination rule for each logical operator. They introduce a logical operator in the conclusion or remove a logical operator from a premise of the rule. Some of the rules allow an assumption of a certain type to be discharged. To indicate which assumption is discharged by which inference, we also assign labels to both the assumption and the inference. This is indicated by writing the assumption as “[ $\varphi$ ]<sup>n</sup>.”

It is customary to consider rules for all the logical operators  $\wedge$ ,  $\vee$ ,  $\rightarrow$ ,  $\neg$ , and  $\perp$ , even if some of those are defined.

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## Bibliography