

ntd.1 Quantifier Rules

fol:ntd:qrl:
sec

Rules for \forall

$$\frac{\varphi(a)}{\forall x \varphi(x)} \forall\text{Intro} \qquad \frac{\forall x \varphi(x)}{\varphi(t)} \forall\text{Elim}$$

In the rules for \forall , t is a ground term (a term that does not contain any variables), and a is a **constant symbol** which does not occur in the conclusion $\forall x \varphi(x)$, or in any assumption which is **undischarged** in the **derivation** ending with the premise $\varphi(a)$. We call a the *eigenvariable* of the $\forall\text{Intro}$ inference.

Rules for \exists

$$\frac{\varphi(t)}{\exists x \varphi(x)} \exists\text{Intro} \qquad \frac{\begin{array}{c} [\varphi(a)]^n \\ \vdots \\ \vdots \\ \vdots \\ \chi \end{array}}{\chi} \exists\text{Elim}$$

Again, t is a ground term, and a is a constant which does not occur in the premise $\exists x \varphi(x)$, in the conclusion χ , or any assumption which is **undischarged** in the **derivations** ending with the two premises (other than the assumptions $\varphi(a)$). We call a the *eigenvariable* of the $\exists\text{Elim}$ inference.

The condition that an eigenvariable neither occur in the premises nor in any assumption that is **undischarged** in the **derivations** leading to the premises for the $\forall\text{Intro}$ or $\exists\text{Elim}$ inference is called the *eigenvariable condition*.

We use the term “eigenvariable” even though a in the above rules is a constant. This has historical reasons. [explanation](#)

In $\exists\text{Intro}$ and $\forall\text{Elim}$ there are no restrictions, and the term t can be anything, so we do not have to worry about any conditions. On the other hand, in the $\exists\text{Elim}$ and $\forall\text{Intro}$ rules, the eigenvariable condition requires that the **constant symbol** a does not occur anywhere in the conclusion or in an **undischarged** assumption. The condition is necessary to ensure that the system is sound, i.e., only **derives sentences** from **undischarged** assumptions from which the follow. Without this condition, the following would be allowed:

$$\frac{\frac{\exists x \varphi(x) \quad \frac{[\varphi(a)]^1}{\forall x \varphi(x)} * \forall \text{Intro}}{\forall x \varphi(x)} \quad \exists \text{Elim}}{\exists x \varphi(x)}$$

However, $\exists x \varphi(x) \not\equiv \forall x \varphi(x)$.

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Bibliography