

ntd.1 Derivability and the Quantifiers

fol:ntd:qpr:

fol:ntd:qpr:^{sec}

thm:strong-generalization

Theorem ntd.1. *If c is a constant not occurring in Γ or $\varphi(x)$ and $\Gamma \vdash \varphi(c)$, then $\Gamma \vdash \forall x \varphi(x)$.*

Proof. Let δ be a **derivation** of $\varphi(c)$ from Γ . By adding a \forall Intro inference, we obtain a proof of $\forall x \varphi(x)$. Since c does not occur in Γ or $\varphi(x)$, the eigenvariable condition is satisfied. \square

fol:ntd:qpr:

prop:provability-quantifiers

Proposition ntd.2.

1. $\varphi(t) \vdash \exists x \varphi(x)$.
2. $\forall x \varphi(x) \vdash \varphi(t)$.

Proof. 1. The following is a **derivation** of $\exists x \varphi(x)$ from $\varphi(t)$:

$$\frac{\varphi(t)}{\exists x \varphi(x)} \exists\text{Intro}$$

2. The following is a **derivation** of $\varphi(t)$ from $\forall x \varphi(x)$:

$$\frac{\forall x \varphi(x)}{\varphi(t)} \forall\text{Elim}$$

\square

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Bibliography