ntd.1 Derivability and the Quantifiers

fol:ntd:qpr:
sec

fol:ntd:qpr:
thm:strong-generalization

Theorem ntd.1. *If c is a constant not occurring in Γ or $\varphi(x)$ and $\Gamma \vdash \varphi(c)$, then $\Gamma \vdash \forall x\, \varphi(x)$.*

Proof. Let δ be a derivation of $\varphi(c)$ from Γ. By adding a $\forall$Intro inference, we obtain a proof of $\forall x\, \varphi(x)$. Since c does not occur in Γ or $\varphi(x)$, the eigenvariable condition is satisfied. □

fol:ntd:qpr:
prop:provability-quantifiers

Proposition ntd.2.

1. $\varphi(t) \vdash \exists x\, \varphi(x)$.
2. $\forall x\, \varphi(x) \vdash \varphi(t)$.

Proof. 1. The following is a derivation of $\exists x\, \varphi(x)$ from $\varphi(t)$:

$$\frac{\varphi(t)}{\exists x\, \varphi(x)}\;\exists\mathrm{Intro}$$

2. The following is a derivation of $\varphi(t)$ from $\forall x\, \varphi(x)$:

$$\frac{\forall x\, \varphi(x)}{\varphi(t)}\;\forall\mathrm{Elim}$$

□

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Bibliography