

## ntd.1 Derivability and the Quantifiers

fol:ntd:qpr:  
sec

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thm:strong-generalization **Theorem ntd.1.** *If  $c$  is a constant not occurring in  $\Gamma$  or  $\varphi(x)$  and  $\Gamma \vdash \varphi(c)$ , then  $\Gamma \vdash \forall x \varphi(x)$ .*

*Proof.* Let  $\delta$  be a **derivation** of  $\varphi(c)$  from  $\Gamma$ . By adding a  $\forall$ Intro inference, we obtain a proof of  $\forall x \varphi(x)$ . Since  $c$  does not occur in  $\Gamma$  or  $\varphi(x)$ , the eigenvariable condition is satisfied.  $\square$

fol:ntd:qpr:  
prop:provability-quantifiers

**Proposition ntd.2.**

1.  $\varphi(t) \vdash \exists x \varphi(x)$ .
2.  $\forall x \varphi(x) \vdash \varphi(t)$ .

*Proof.* 1. The following is a **derivation** of  $\exists x \varphi(x)$  from  $\varphi(t)$ :

$$\frac{\varphi(t)}{\exists x \varphi(x)} \exists\text{Intro}$$

2. The following is a **derivation** of  $\varphi(t)$  from  $\forall x \varphi(x)$ :

$$\frac{\forall x \varphi(x)}{\varphi(t)} \forall\text{Elim}$$

$\square$

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## Bibliography