

ntd.1 Derivability and the Quantifiers

fol:ntd:qpr:
sec The completeness theorem also requires that the natural deduction rules yield explanation the facts about \vdash established in this section.

fol:ntd:qpr:
thm:strong-generalization **Theorem ntd.1.** *If c is a constant not occurring in Γ or $\varphi(x)$ and $\Gamma \vdash \varphi(c)$, then $\Gamma \vdash \forall x \varphi(x)$.*

Proof. Let δ be a **derivation** of $\varphi(c)$ from Γ . By adding a \forall Intro inference, we obtain a **derivation** of $\forall x \varphi(x)$. Since c does not occur in Γ or $\varphi(x)$, the eigenvariable condition is satisfied. \square

fol:ntd:qpr:
prop:provability-quantifiers **Proposition ntd.2.**

1. $\varphi(t) \vdash \exists x \varphi(x)$.
2. $\forall x \varphi(x) \vdash \varphi(t)$.

Proof. 1. The following is a **derivation** of $\exists x \varphi(x)$ from $\varphi(t)$:

$$\frac{\varphi(t)}{\exists x \varphi(x)} \exists\text{Intro}$$

2. The following is a **derivation** of $\varphi(t)$ from $\forall x \varphi(x)$:

$$\frac{\forall x \varphi(x)}{\varphi(t)} \forall\text{Elim} \quad \square$$

Photo Credits

Bibliography