The completeness theorem also requires that the natural deduction rules yield the facts about $\vdash$ established in this section.

**Theorem ntd.1.** If $c$ is a constant not occurring in $\Gamma$ or $\varphi(x)$ and $\Gamma \vdash \varphi(c)$, then $\Gamma \vdash \forall x \varphi(x)$.

*Proof.* Let $\delta$ be a derivation of $\varphi(c)$ from $\Gamma$. By adding a $\forall$Intro inference, we obtain a derivation of $\forall x \varphi(x)$. Since $c$ does not occur in $\Gamma$ or $\varphi(x)$, the eigenvariable condition is satisfied. \hfill $\square$

**Proposition ntd.2.**

1. $\varphi(t) \vdash \exists x \varphi(x)$.
2. $\forall x \varphi(x) \vdash \varphi(t)$.

*Proof.* 1. The following is a derivation of $\exists x \varphi(x)$ from $\varphi(t)$:

$$
\frac{\varphi(t)}{\exists x \varphi(x)} \exists\text{Intro}
$$

2. The following is a derivation of $\varphi(t)$ from $\forall x \varphi(x)$:

$$
\frac{\forall x \varphi(x)}{\varphi(t)} \forall\text{Elim}
$$

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