

ntd.1 Derivability and the Propositional Connectives

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Proposition ntd.1.

1. Both $\varphi \wedge \psi \vdash \varphi$ and $\varphi \wedge \psi \vdash \psi$.
2. $\varphi, \psi \vdash \varphi \wedge \psi$.

Proof. 1. We can **derive** both

$$\frac{\varphi \wedge \psi}{\varphi} \wedge\text{Elim} \qquad \frac{\varphi \wedge \psi}{\psi} \wedge\text{Elim}$$

2. We can **derive**:

$$\frac{\varphi \quad \psi}{\varphi \wedge \psi} \wedge\text{Intro}$$

□

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Proposition ntd.2.

1. $\varphi \vee \psi, \neg\varphi, \neg\psi$ is inconsistent.
2. Both $\varphi \vdash \varphi \vee \psi$ and $\psi \vdash \varphi \vee \psi$.

Proof. 1. Consider the following **derivation**:

$$\frac{1 \quad \varphi \vee \psi \quad \frac{\neg\varphi \quad [\varphi]^1}{\perp} \neg\text{Elim} \quad \frac{\neg\psi \quad [\psi]^1}{\perp} \neg\text{Elim}}{\perp} \vee\text{Elim}$$

This is a **derivation** of \perp from **undischarged** assumptions $\varphi \vee \psi$, $\neg\varphi$, and $\neg\psi$.

2. We can **derive** both

$$\frac{\varphi}{\varphi \vee \psi} \vee\text{Intro} \qquad \frac{\psi}{\varphi \vee \psi} \vee\text{Intro}$$

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Proposition ntd.3.

1. $\varphi, \varphi \rightarrow \psi \vdash \psi$.
2. Both $\neg\varphi \vdash \varphi \rightarrow \psi$ and $\psi \vdash \varphi \rightarrow \psi$.

Proof. 1. We can **derive**:

$$\frac{\varphi \rightarrow \psi \quad \psi}{\psi} \rightarrow\text{Elim}$$

2. This is shown by the following two **derivations**:

$$\frac{\frac{\frac{\neg\varphi \quad [\varphi]^1}{\perp} \neg\text{Elim} \quad \perp}{\psi} \perp_I}{\varphi \rightarrow \psi} \rightarrow\text{Intro} \quad \frac{\psi}{\varphi \rightarrow \psi} \rightarrow\text{Intro}$$

Note that $\rightarrow\text{Intro}$ may, but does not have to, **discharge** the assumption φ . \square

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Bibliography