ntd.1 Propositional Rules

Rules for $\land$

- $\varphi \land \psi \vdash \varphi \land \psi$ (\textit{$\land$Intro})
- $\varphi \land \psi \vdash \varphi$ (\textit{$\land$Elim})
- $\varphi \land \psi \vdash \psi$ (\textit{$\land$Elim})

Rules for $\lor$

- $\varphi \lor \psi \vdash \varphi$ (\textit{$\lor$Intro})
- $\varphi \lor \psi \vdash \psi$ (\textit{$\lor$Intro})
- $\varphi \lor \psi \vdash \chi$ (\textit{$\lor$Elim})

Rules for $\rightarrow$

- $\varphi \vdash \psi$ (\textit{$\rightarrow$Intro})
- $\varphi \rightarrow \psi \vdash \varphi$ (\textit{$\rightarrow$Elim})

Rules for $\neg$

- $\neg \varphi \vdash \neg \varphi$ (\textit{$\neg$Intro})
- $\neg \varphi \vdash \bot$ (\textit{$\neg$Elim})
Note that ¬Intro and ⊥C are very similar: The difference is that ¬Intro derives a negated sentence ¬ϕ but ⊥C a positive sentence ϕ.

Whenever a rule indicates that some assumption may be discharged, we take this to be a permission, but not a requirement. E.g., in the →Intro rule, we may discharge any number of assumptions of the form ϕ in the derivation of the premise ψ, including zero.