

## ntd.1 Derivations

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sec We've said what an assumption is, and we've given the rules of inference. explanation **Derivations** in the sequent calculus are inductively generated from these: each **derivation** either is an assumption on its own, or consists of one, two, or three **derivations** followed by a correct inference.

**Definition ntd.1 (Derivation).** A *derivation* of a sentence  $\varphi$  from assumptions  $\Gamma$  is a tree of **sentences** satisfying the following conditions:

1. The topmost **sentences** of the tree are either in  $\Gamma$  or are **discharged** by an inference in the tree.
2. The bottommost **sentence** of the tree is  $\varphi$ .
3. Every **sentence** in the tree except  $\varphi$  is a premise of a correct application of an inference rule whose conclusion stands directly below that **sentence** in the tree.

We then say that  $\varphi$  is the *conclusion* of the **derivation** and that  $\varphi$  is *derivable* from  $\Gamma$ .

**Example ntd.2.** Every assumption on its own is a **derivation**. So, e.g.,  $\chi$  by itself is a **derivation**, and so is  $\theta$  by itself. We can obtain a new **derivation** from these by applying, say, the  $\wedge$ Intro rule,

$$\frac{\varphi \quad \psi}{\varphi \wedge \psi} \wedge\text{Intro}$$

These rules are meant to be general: we can replace the  $\varphi$  and  $\psi$  in it with any **sentences**, e.g., by  $\chi$  and  $\theta$ . Then the conclusion would be  $\chi \wedge \theta$ , and so

$$\frac{\chi \quad \theta}{\chi \wedge \theta} \wedge\text{Intro}$$

is a correct **derivation**. Of course, we can also switch the assumptions, so that  $\theta$  plays the role of  $\varphi$  and  $\chi$  that of  $\psi$ . Thus,

$$\frac{\theta \quad \chi}{\theta \wedge \chi} \wedge\text{Intro}$$

is also a correct derivation.

We can now apply another rule, say,  $\rightarrow$ Intro, which allows us to conclude a conditional and allows us to **discharge** any assumption that is identical to the conclusion of that conditional. So both of the following would be correct **derivations**:

$$1 \frac{\frac{[\chi]^1 \quad \theta}{\chi \wedge \theta} \wedge\text{Intro}}{\chi \rightarrow (\chi \wedge \theta)} \rightarrow\text{Intro} \quad 1 \frac{\frac{\chi \quad [\theta]^1}{\chi \wedge \theta} \wedge\text{Intro}}{\theta \rightarrow (\chi \wedge \theta)} \rightarrow\text{Intro}$$

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**Bibliography**