int.1 Syntax

fol:int:syn:sec We first must make precise what strings of symbols count as sentences of first-order logic. We'll do this later; for now we'll just proceed by example. The basic building blocks—the vocabulary—of first-order logic divides into two parts. The first part is the symbols we use to say specific things or to pick out specific things. We pick out things using constant symbols, and we say stuff about the things we pick out using predicate symbols. E.g, we might use a as a constant symbol to pick out a single thing, and then say something about it using the sentence $P(a)$. If you have meanings for "a" and "P" in mind, you can read $P(a)$ as a sentence of English (and you probably have done so when you first learned formal logic). Once you have such simple sentences of first-order logic, you can build more complex ones using the second part of the vocabulary: the logical symbols (connectives and quantifiers). So, for instance, we can form expressions like $(P(a) \land Q(b))$ or $\exists x\, P(x)$.

In order to provide the precise definitions of semantics and the rules of our derivation systems required for rigorous meta-logical study, we first of all have to give a precise definition of what counts as a sentence of first-order logic. The basic idea is easy enough to understand: there are some simple sentences we can form from just predicate symbols and constant symbols, such as $P(a)$. And then from these we form more complex ones using the connectives and quantifiers. But what exactly are the rules by which we are allowed to form more complex sentences? These must be specified, otherwise we have not defined "sentence of first-order logic" precisely enough. There are a few issues. The first one is to get the right strings to count as sentences. The second one is to do this in such a way that we can give mathematical proofs about *all* sentences. Finally, we'll have to also give precise definitions of some rudimentary operations with sentences, such as "replace every x in φ by b." The trouble is that the quantifiers and variables we have in first-order logic make it not entirely obvious how this should be done. E.g., should $\exists x\, P(a)$ count as a sentence? What about $\exists x\, \exists x\, P(x)$? What should the result of "replace x by b in $(P(x) \land \exists x\, P(x))$" be?

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Bibliography