

axd.1 Derivations with Quantifiers

fol:axd:prq:
sec

Example axd.1. Let us give a derivation of $(\forall x \varphi(x) \wedge \forall y \psi(y)) \rightarrow \forall x (\varphi(x) \wedge \psi(x))$.

First, note that

$$(\forall x \varphi(x) \wedge \forall y \psi(y)) \rightarrow \forall x \varphi(x)$$

is an instance of ??, and

$$\forall x \varphi(x) \rightarrow \varphi(a)$$

of ?. So, by ?, we know that

$$(\forall x \varphi(x) \wedge \forall y \psi(y)) \rightarrow \varphi(a)$$

is derivable. Likewise, since

$$\begin{aligned} (\forall x \varphi(x) \wedge \forall y \psi(y)) \rightarrow \forall y \psi(y) & \quad \text{and} \\ \forall y \psi(y) \rightarrow \psi(a) & \end{aligned}$$

are instances of ? and ?, respectively,

$$(\forall x \varphi(x) \wedge \forall y \psi(y)) \rightarrow \psi(a)$$

is derivable by ?. Using an appropriate instance of ? and two applications of MP, we see that

$$(\forall x \varphi(x) \wedge \forall y \psi(y)) \rightarrow (\varphi(a) \wedge \psi(a))$$

is derivable. We can now apply QR to obtain

$$(\forall x \varphi(x) \wedge \forall y \psi(y)) \rightarrow \forall x (\varphi(x) \wedge \psi(x)).$$

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Bibliography