

## axd.1 Derivability and the Quantifiers

fol:axd:qpr:

fol:axd:qpr:<sup>sec</sup>

thm:strong-generalization

**Theorem axd.1.** *If  $c$  is a constant symbol not occurring in  $\Gamma$  or  $\varphi(x)$  and  $\Gamma \vdash \varphi(c)$ , then  $\Gamma \vdash \forall x \varphi(x)$ .*

*Proof.* By the deduction theorem,  $\Gamma \vdash \top \rightarrow \varphi(c)$ . Since  $c$  does not occur in  $\Gamma$  or  $\top$ , we get  $\Gamma \vdash \top \rightarrow \varphi(c)$ . By the deduction theorem again,  $\Gamma \vdash \forall x \varphi(x)$ .  $\square$

fol:axd:qpr:

prop:provability-quantifiers

**Proposition axd.2.**

1.  $\varphi(t) \vdash \exists x \varphi(x)$ .

2.  $\forall x \varphi(x) \vdash \varphi(t)$ .

*Proof.* 1. By ?? and the deduction theorem.

2. By ?? and the deduction theorem.

$\square$

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## Bibliography