axd.1 Derivability and the Quantifiers

The completeness theorem also requires that axiomatic deductions yield the facts about \( \vdash \) established in this section.

**Theorem axd.1.** If \( c \) is a constant symbol not occurring in \( \Gamma \) or \( \varphi(x) \) and \( \Gamma \vdash \varphi(c) \), then \( \Gamma \vdash \forall x \varphi(x) \).

*Proof.* By the deduction theorem, \( \Gamma \vdash \top \rightarrow \varphi(c) \). Since \( c \) does not occur in \( \Gamma \) or \( \top \), we get \( \Gamma \vdash \top \rightarrow \varphi(c) \). By the deduction theorem again, \( \Gamma \vdash \forall x \varphi(x) \). \( \square \)

**Proposition axd.2.**

1. \( \varphi(t) \vdash \exists x \varphi(x) \).
2. \( \forall x \varphi(x) \vdash \varphi(t) \).

*Proof.*

1. By ?? and the deduction theorem.
2. By ?? and the deduction theorem. \( \square \)

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Bibliography