axd.1 Derivability and the Propositional Connectives

fol:axd:ppr:sec

Proposition axd.1.

fol:axd:ppr:prop:provability-land

fol:axd:ppr:prop:provability-land-left

fol:axd:ppr:prop:provability-land-right

1. *Both* $\varphi \land \psi \vdash \varphi$ *and* $\varphi \land \psi \vdash \psi$
2. $\varphi, \psi \vdash \varphi \land \psi$.

Proof. 1. From **??** and **??** by modus ponens.

2. From **??** by two applications of modus ponens.

□

Proposition axd.2.

fol:axd:ppr:prop:provability-lor

1. $\varphi \lor \psi, \neg\varphi, \neg\psi$ *is inconsistent.*
2. *Both* $\varphi \vdash \varphi \lor \psi$ *and* $\psi \vdash \varphi \lor \psi$.

Proof. 1. From **??** we get $\vdash \neg\varphi \to (\varphi \to \bot)$ and $\vdash \neg\varphi \to (\varphi \to \bot)$. So by the deduction theorem, we have $\{\neg\varphi\} \vdash \varphi \to \bot$ and $\{\neg\psi\} \vdash \psi \to \bot$. From **??** we get $\{\neg\varphi, \neg\psi\} \vdash (\varphi \lor \psi) \to \bot$. By the deduction theorem, $\{\varphi \lor \psi, \neg\varphi, \neg\psi\} \vdash \bot$.

2. From **??** and **??** by modus ponsens.

□

Proposition axd.3.

fol:axd:ppr:prop:provability-lif

fol:axd:ppr:prop:provability-lif-left

fol:axd:ppr:prop:provability-lif-right

1. $\varphi, \varphi \to \psi \vdash \psi$.
2. *Both* $\neg\varphi \vdash \varphi \to \psi$ *and* $\psi \vdash \varphi \to \psi$.

Proof. 1. We can derive:

1.	φ	Hyp
2.	$\varphi \to \psi$	Hyp
3.	ψ	1, 2, mp

2. By **??** and **??** and the deduction theorem, respectively.

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Bibliography