

axd.1 Derivability and the Propositional Connectives

fol:axd:ppr:
sec

fol:axd:ppr: **Proposition axd.1.**

prop:provability-land

fol:axd:ppr:

prop:provability-land-left

fol:axd:ppr:

prop:provability-land-right

1. Both $\varphi \wedge \psi \vdash \varphi$ and $\varphi \wedge \psi \vdash \psi$

2. $\varphi, \psi \vdash \varphi \wedge \psi$.

Proof. 1. From ?? and ?? by modus ponens.

2. From ?? by two applications of modus ponens. □

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prop:provability-lor

Proposition axd.2.

1. $\varphi \vee \psi, \neg\varphi, \neg\psi$ is inconsistent.

2. Both $\varphi \vdash \varphi \vee \psi$ and $\psi \vdash \varphi \vee \psi$.

Proof. 1. From ?? we get $\vdash \neg\varphi \rightarrow (\varphi \rightarrow \perp)$ and $\vdash \neg\psi \rightarrow (\psi \rightarrow \perp)$. So by the deduction theorem, we have $\{\neg\varphi\} \vdash \varphi \rightarrow \perp$ and $\{\neg\psi\} \vdash \psi \rightarrow \perp$.

From ?? we get $\{\neg\varphi, \neg\psi\} \vdash (\varphi \vee \psi) \rightarrow \perp$. By the deduction theorem, $\{\varphi \vee \psi, \neg\varphi, \neg\psi\} \vdash \perp$.

2. From ?? and ?? by modus ponens. □

fol:axd:ppr:

prop:provability-lif

Proposition axd.3.

fol:axd:ppr:

prop:provability-lif-left

1. $\varphi, \varphi \rightarrow \psi \vdash \psi$.

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prop:provability-lif-right

2. Both $\neg\varphi \vdash \varphi \rightarrow \psi$ and $\psi \vdash \varphi \rightarrow \psi$.

Proof. 1. We can derive:

1. φ HYP
2. $\varphi \rightarrow \psi$ HYP
3. ψ 1, 2, MP

2. By ?? and ?? and the deduction theorem, respectively. □

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Bibliography