Derivations with Identity predicate

In order to accommodate $=$ in derivations, we simply add new axiom schemas. The definition of derivation and $\vdash$ remains the same, we just also allow the new axioms.

**Definition axd.1** (Axioms for identity predicate).

\[
\begin{align*}
\text{ax:id1} & : t = t, & (1) \\
\text{ax:id2} & : t_1 = t_2 \rightarrow (\psi(t_1) \rightarrow \psi(t_2)), & (2)
\end{align*}
\]

for any ground terms $t, t_1, t_2$.

**Proposition axd.2.** The axioms eq. (1) and eq. (2) are valid.

**Proof.** Exercise.

**Problem axd.1.** Prove Proposition axd.2.

**Proposition axd.3.** $\Gamma \vdash t = t$, for any term $t$ and set $\Gamma$.

**Proposition axd.4.** If $\Gamma \vdash \varphi(t_1)$ and $\Gamma \vdash t_1 = t_2$, then $\Gamma \vdash \varphi(t_2)$.

**Proof.** The formula

\[(t_1 = t_2 \rightarrow (\varphi(t_1) \rightarrow \varphi(t_2)))\]

is an instance of eq. (2). The conclusion follows by two applications of MP.

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Bibliography