

axd.1 Derivations with Identity predicate

fol:axd:ide: In order to accommodate = in **derivations**, we simply add new axiom schemas.
sec The definition of **derivation** and \vdash remains the same, we just also allow the new axioms.

Definition axd.1 (Axioms for **identity predicate**).

fol:axd:ide: $t = t,$ (1)

ax:id1
fol:axd:ide: $t_1 = t_2 \rightarrow (\psi(t_1) \rightarrow \psi(t_2)),$ (2)
ax:id2

for any ground terms t, t_1, t_2 .

fol:axd:ide: **Proposition axd.2.** *The axioms eq. (1) and eq. (2) are valid.*
prop:sound

Proof. Exercise. □

Problem axd.1. Prove **Proposition axd.2**.

fol:axd:ide: **Proposition axd.3.** $\Gamma \vdash t = t$, for any term t and set Γ .
prop:iden1

fol:axd:ide: **Proposition axd.4.** If $\Gamma \vdash \varphi(t_1)$ and $\Gamma \vdash t_1 = t_2$, then $\Gamma \vdash \varphi(t_2)$.
prop:iden2

Proof. The **formula**

$$(t_1 = t_2 \rightarrow (\varphi(t_1) \rightarrow \varphi(t_2)))$$

is an instance of **eq. (2)**. The conclusion follows by two applications of MP. □

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Bibliography