axd.1 Derivations with Identity predicate

fol:axd:ide: sec

In order to accommodate $=$ in derivations, we simply add new axiom schemas. The definition of derivation and $\vdash$ remains the same, we just also allow the new axioms.

Definition axd.1 (Axioms for identity predicate)**.**

fol:axd:ide: ax:id1

$$t = t, \tag{1}$$

fol:axd:ide: ax:id2

$$t_1 = t_2 \rightarrow (\psi(t_1) \rightarrow \psi(t_2)), \tag{2}$$

for any ground terms t, t_1, t_2.

fol:axd:ide: prop:sound

Proposition axd.2. *The axioms eq. (1) and eq. (2) are valid.*

Proof. Exercise. □

Problem axd.1. Prove Proposition axd.2.

fol:axd:ide: prop:iden1

Proposition axd.3. *$\Gamma \vdash t = t$, for any term t and set Γ.*

fol:axd:ide: prop:iden2

Proposition axd.4. *If $\Gamma \vdash \varphi(t_1)$ and $\Gamma \vdash t_1 = t_2$, then $\Gamma \vdash \varphi(t_2)$.*

Proof. The formula

$$(t_1 = t_2 \rightarrow (\varphi(t_1) \rightarrow \varphi(t_2)))$$

is an instance of eq. (2). The conclusion follows by two applications of MP. □

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Bibliography