

axd.1 Derivations with Identity predicate

fol:axd:ide: sec In order to accommodate = in **derivations**, we simply add new axiom schemas. The definition of **derivation** and \vdash remains the same, we just also allow the new axioms.

Definition axd.1 (Axioms for identity predicate).

fol:axd:ide: ax:id1
$$t = t, \tag{1}$$

fol:axd:ide: ax:id2
$$t_1 = t_2 \rightarrow (\psi(t_1) \rightarrow \psi(t_2)), \tag{2}$$

for any closed terms t, t_1, t_2 .

fol:axd:ide: prop:sound **Proposition axd.2.** *The axioms eq. (1) and eq. (2) are valid.*

Proof. Exercise. □

Problem axd.1. Prove **Proposition axd.2**.

fol:axd:ide: prop:iden1 **Proposition axd.3.** $\Gamma \vdash t = t$, for any term t and set Γ .

fol:axd:ide: prop:iden2 **Proposition axd.4.** If $\Gamma \vdash \varphi(t_1)$ and $\Gamma \vdash t_1 = t_2$, then $\Gamma \vdash \varphi(t_2)$.

Proof. The **formula**
$$(t_1 = t_2 \rightarrow (\varphi(t_1) \rightarrow \varphi(t_2)))$$

is an instance of **eq. (2)**. The conclusion follows by two applications of MP. □

Photo Credits

Bibliography