axd.1  Axioms and Rules for Quantifiers

Definition axd.1 (Axioms for quantifiers). The axioms governing quantifiers are all instances of the following:

\begin{align*}
\forall x \psi & \rightarrow \psi(t), \quad (1) \\
\psi(t) & \rightarrow \exists x \psi, \quad (2)
\end{align*}

for any closed term \( t \).

Definition axd.2 (Rules for quantifiers).
If \( \psi \rightarrow \varphi(a) \) already occurs in the derivation and \( a \) does not occur in \( \Gamma \) or \( \psi \), then \( \psi \rightarrow \forall x \varphi(x) \) is a correct inference step.

If \( \varphi(a) \rightarrow \psi \) already occurs in the derivation and \( a \) does not occur in \( \Gamma \) or \( \psi \), then \( \exists x \varphi(x) \rightarrow \psi \) is a correct inference step.

We’ll abbreviate either of these by “QR.”

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Bibliography