

## axd.1 Axiom and Rules for the Propositional Connectives

fol:axd:prp:  
sec

**Definition axd.1** (Axioms). The set of  $Ax_0$  of *axioms* for the propositional connectives comprises all **formulas** of the following forms:

fol:axd:prp:  $(\varphi \wedge \psi) \rightarrow \varphi$  (1)

ax:land1  
fol:axd:prp:  $(\varphi \wedge \psi) \rightarrow \psi$  (2)

ax:land2  
fol:axd:prp:  $\varphi \rightarrow (\psi \rightarrow (\varphi \wedge \psi))$  (3)

ax:land3  
fol:axd:prp:  $\varphi \rightarrow (\varphi \vee \psi)$  (4)

ax:lor1  
fol:axd:prp:  $\varphi \rightarrow (\psi \vee \varphi)$  (5)

ax:lor2  
fol:axd:prp:  $(\varphi \rightarrow \chi) \rightarrow ((\psi \rightarrow \chi) \rightarrow ((\varphi \vee \psi) \rightarrow \chi))$  (6)

ax:lor3  
fol:axd:prp:  $\varphi \rightarrow (\psi \rightarrow \varphi)$  (7)

ax:lif1  
fol:axd:prp:  $(\varphi \rightarrow (\psi \rightarrow \chi)) \rightarrow ((\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow \chi))$  (8)

ax:lif2  
fol:axd:prp:  $(\varphi \rightarrow \psi) \rightarrow ((\varphi \rightarrow \neg\psi) \rightarrow \neg\varphi)$  (9)

ax:lnot1  
fol:axd:prp:  $\neg\varphi \rightarrow (\varphi \rightarrow \psi)$  (10)

ax:lnot2  
fol:axd:prp:  $\top$  (11)

ax:ltrue  
fol:axd:prp:  $\perp \rightarrow \varphi$  (12)

ax:lfalse1  
fol:axd:prp:  $(\varphi \rightarrow \perp) \rightarrow \neg\varphi$  (13)

ax:lfalse2  
fol:axd:prp:  $\neg\neg\varphi \rightarrow \varphi$  (14)

ax:dne

**Definition axd.2** (Modus ponens). If  $\psi$  and  $\psi \rightarrow \varphi$  already occur in a derivation, then  $\varphi$  is a correct inference step.

We'll abbreviate the rule modus ponens as “MP.”

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## Bibliography