axd.1 Axiom and Rules for the Propositional Connectives

fol:axd:prp:sec

Definition axd.1 (Axioms)**.** The set of Ax_0 of *axioms* for the propositional connectives comprises all formulas of the following forms:

$$(\varphi \wedge \psi) \to \varphi \tag{1}$$

$$(\varphi \wedge \psi) \to \psi \tag{2}$$

$$\varphi \to (\psi \to (\varphi \wedge \psi)) \tag{3}$$

$$\varphi \to (\varphi \vee \psi) \tag{4}$$

$$\varphi \to (\psi \vee \varphi) \tag{5}$$

$$(\varphi \to \chi) \to ((\psi \to \chi) \to ((\varphi \vee \psi) \to \chi)) \tag{6}$$

$$\varphi \to (\psi \to \varphi) \tag{7}$$

$$(\varphi \to (\psi \to \chi)) \to ((\varphi \to \psi) \to (\varphi \to \chi)) \tag{8}$$

$$(\varphi \to \psi) \to ((\varphi \to \neg\psi) \to \neg\varphi) \tag{9}$$

$$\neg\varphi \to (\varphi \to \psi) \tag{10}$$

$$\top \tag{11}$$

$$\bot \to \varphi \tag{12}$$

$$(\varphi \to \bot) \to \neg\varphi \tag{13}$$

$$\neg\neg\varphi \to \varphi \tag{14}$$

Definition axd.2 (Modus ponens)**.** If ψ and $\psi \to \varphi$ already occur in a derivation, then φ is a correct inference step.

We'll abbreviate the rule modus ponens as "MP."

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Bibliography