

axd.1 Axiom and Rules for the Propositional Connectives

fol:axd:prp:
sec

Definition axd.1 (Axioms). The set of Ax_0 of *axioms* for the propositional connectives comprises all **formulas** of the following forms:

fol:axd:prp: $(\varphi \wedge \psi) \rightarrow \varphi$ (1)

ax:land1
fol:axd:prp: $(\varphi \wedge \psi) \rightarrow \psi$ (2)

ax:land2
fol:axd:prp: $\varphi \rightarrow (\psi \rightarrow (\varphi \wedge \psi))$ (3)

ax:land3
fol:axd:prp: $\varphi \rightarrow (\varphi \vee \psi)$ (4)

ax:lor1
fol:axd:prp: $\varphi \rightarrow (\psi \vee \varphi)$ (5)

ax:lor2
fol:axd:prp: $(\varphi \rightarrow \chi) \rightarrow ((\psi \rightarrow \chi) \rightarrow ((\varphi \vee \psi) \rightarrow \chi))$ (6)

ax:lor3
fol:axd:prp: $\varphi \rightarrow (\psi \rightarrow \varphi)$ (7)

ax:lif1
fol:axd:prp: $(\varphi \rightarrow (\psi \rightarrow \chi)) \rightarrow ((\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow \chi))$ (8)

ax:lif2
fol:axd:prp: $(\varphi \rightarrow \psi) \rightarrow ((\varphi \rightarrow \neg\psi) \rightarrow \neg\varphi)$ (9)

ax:lnot1
fol:axd:prp: $\neg\varphi \rightarrow (\varphi \rightarrow \psi)$ (10)

ax:lnot2
fol:axd:prp: \top (11)

ax:ltrue
fol:axd:prp: $\perp \rightarrow \varphi$ (12)

ax:lfalse1
fol:axd:prp: $(\varphi \rightarrow \perp) \rightarrow \neg\varphi$ (13)

ax:lfalse2
fol:axd:prp: $\neg\neg\varphi \rightarrow \varphi$ (14)

ax:dne

Definition axd.2 (Modus ponens). If ψ and $\psi \rightarrow \varphi$ already occur in a derivation, then φ is a correct inference step.

We'll abbreviate the rule modus ponens as “MP.”

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Bibliography