

axd.1 Axiom and Rules for the Propositional Connectives

fol:axd:prp:
sec

Definition axd.1 (Axioms). The set of Ax_0 of *axioms* for the propositional connectives comprises all **formulas** of the following forms:

fol:axd:prp:	$(\varphi \wedge \psi) \rightarrow \varphi$	(1)
ax:land1 fol:axd:prp:	$(\varphi \wedge \psi) \rightarrow \psi$	(2)
ax:land2 fol:axd:prp:	$\varphi \rightarrow (\psi \rightarrow (\varphi \wedge \psi))$	(3)
ax:land3 fol:axd:prp:	$\varphi \rightarrow (\varphi \vee \psi)$	(4)
ax:lor1 fol:axd:prp:	$\varphi \rightarrow (\psi \vee \varphi)$	(5)
ax:lor2 fol:axd:prp:	$(\varphi \rightarrow \chi) \rightarrow ((\psi \rightarrow \chi) \rightarrow ((\varphi \vee \psi) \rightarrow \chi))$	(6)
ax:lor3 fol:axd:prp:	$\varphi \rightarrow (\psi \rightarrow \varphi)$	(7)
ax:lif1 fol:axd:prp:	$(\varphi \rightarrow (\psi \rightarrow \chi)) \rightarrow ((\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow \chi))$	(8)
ax:lif2 fol:axd:prp:	$(\varphi \rightarrow \psi) \rightarrow ((\varphi \rightarrow \neg\psi) \rightarrow \neg\varphi)$	(9)
ax:lnot1 fol:axd:prp:	$\neg\varphi \rightarrow (\varphi \rightarrow \psi)$	(10)
ax:lnot2 fol:axd:prp:	\top	(11)
ax:ltrue fol:axd:prp:	$\perp \rightarrow \varphi$	(12)
ax:lfalse1 fol:axd:prp:	$(\varphi \rightarrow \perp) \rightarrow \neg\varphi$	(13)
ax:lfalse2 fol:axd:prp:	$\neg\neg\varphi \rightarrow \varphi$	(14)
ax:dne		

Definition axd.2 (Modus ponens). If ψ and $\psi \rightarrow \varphi$ already occur in a derivation, then φ is a correct inference step.

We'll abbreviate the rule modus ponens as “MP.”

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Bibliography