min.1 Truth and Falsity of Counterfactuals

A counterfactual \( \varphi \rightarrow \psi \) is (non-vacuously) true if the closest \( \varphi \)-worlds are all \( \psi \)-worlds, as depicted in Figure 1. A counterfactual is also true at \( w \) if the system of spheres around \( w \) has no \( \varphi \)-admitting spheres at all. In that case it is vacuously true (see Figure 2).

It can be false in two ways. One way is if the closest \( \varphi \)-worlds are not all \( \psi \)-worlds, but some of them are. In this case, \( \varphi \rightarrow \neg \psi \) is also false (see Figure 3). If the closest \( \varphi \)-worlds do not overlap with the \( \psi \)-worlds at all, then \( \varphi \rightarrow \psi \). But, in this case all the closest \( \varphi \)-worlds are \( \neg \psi \)-worlds, and so \( \varphi \rightarrow \neg \psi \) is true (see Figure 4).

In contrast to the strict conditional, counterfactuals may be contingent. Consider the sphere model in Figure 5. The \( \varphi \)-worlds closest to \( u \) are all \( \psi \)-worlds, so \( \mathcal{M}, u \models \varphi \rightarrow \psi \). But there are \( \varphi \)-worlds closest to \( v \) which are not \( \psi \)-worlds, so \( \mathcal{M}, v \not\models \varphi \rightarrow \psi \).
Figure 3: False counterfactual, false opposite

Figure 4: False counterfactual, true opposite

Figure 5: Contingent counterfactual