min.1 Transitivity

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For the material conditional, the chain rule holds: $\varphi \rightarrow \psi, \psi \rightarrow \chi \vDash \varphi \rightarrow \chi$. In other words, the material conditional is transitive. Is the same true for counterfactuals? Consider the following example due to Stalnaker.

> If J. Edgar Hoover had been born a Russian, he would have been a Communist.
>
> If J. Edgar Hoover were a Communist, he would have been be a traitor.
>
> Therefore, If J. Edgar Hoover had been born a Russian, he would have been be a traitor.

If Hoover had been born (at the same time he actually did), not in the United States, but in Russia, he would have grown up in the Soviet Union and become a Communist (let's assume). So the first premise is true. Likewise, the second premise, considered in isolation is true. The conclusion, however, is false: in all likelihood, Hoover would have been a fervent Communist if he had been born in the USSR, and not been a traitor (to his country). The intuitive assignment of truth values is borne out by the Stalnaker–Lewis account. The closest possible world to ours with the only change being Hoover's place of birth is the one where Hoover grows up to be a good citizen of the USSR. This is the closest possible world where the antecedent of the first premise and of the conclusion is true, and in that world Hoover is a loyal member of the Communist party, and so not a traitor. To evaluate the second premise, we have to look at a different world, however: the closest world where Hoover is a Communist, which is one where he was born in the United States, turned, and thus became a traitor.[1]

Problem min.1. Find a convincing, intuitive example for the failure of transitivity of counterfactuals.

cnt:min:tra: ex:trans-counterex

Example min.1. The sphere semantics invalidates the inference, i.e., we have $p \mathrel{\Box\!\!\rightarrow} q, q \mathrel{\Box\!\!\rightarrow} r \nvDash p \mathrel{\Box\!\!\rightarrow} r$. Consider the model $\mathfrak{M} = \langle W, O, V \rangle$ where $W = \{w, w_1, w_2\}$, $O_w = \{\{w\}, \{w, w_1\}, \{w, w_1, w_2\}\}$, $V(p) = \{w_2\}$, $V(q) = \{w_1, w_2\}$, and $V(r) = \{w_1\}$. There is a p-admitting sphere $S = \{w, w_1, w_2\}$ and $p \rightarrow q$ is true at all worlds in it, so $\mathfrak{M}, w \Vdash p \mathrel{\Box\!\!\rightarrow} q$. There is also a q-admitting sphere $S' = \{w, w_1\}$ and $\mathfrak{M} \nVdash q \rightarrow r$ is true at all worlds in it, so $\mathfrak{M}, w \Vdash q \mathrel{\Box\!\!\rightarrow} r$. However, the p-admitting sphere $\{w, w_1, w_2\}$ contains a world, namely w_2, where $\mathfrak{M}, w_2 \nVdash p \rightarrow r$.

Problem min.2. Draw the sphere diagram corresponding to the counterexample in Example min.1.

[1] Of course, to appreciate the force of the example we have to take on board some metaphysical and political assumptions, e.g., that it is possible that Hoover could have been born to Russian parents, or that Communists in the US of the 1950s were traitors to their country.

Problem min.3. In Example min.1, world w_2 is where Hoover is born in Russia, is a communist, and not a traitor, and w_1 is the world where Hoover is born in the US, is a communist, and a traitor. In this model, w_1 is closer to w than w_2 is. Is this necessary? Can you give a counterexample that does not assume that Hoover's being born in Russia is a more remote possibility than him being a Communist?

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Bibliography