



Figure 1: Diagram of a sphere model

con:min:sph:
fig:sphere-model

min.1 Sphere Models

con:min:sph:
sec

One way of providing a formal semantics for counterfactuals is to turn Lewis’s informal account into a mathematical structure. The spheres around a world w then are sets of worlds. Since the spheres are nested, the sets of worlds around w have to be linearly ordered by the subset relation.

Definition min.1. A *sphere model* is a triple $\mathfrak{M} = \langle W, O, V \rangle$ where W is a non-empty set of worlds, $V: \text{At}_0 \rightarrow \wp(W)$ is a valuation, and $O: W \rightarrow \wp(\wp(W))$ assigns to each world w a *system of spheres* O_w . For each w , O_w is a set of sets of worlds, and must satisfy:

1. O_w is *centered* on w : $\{w\} \in O_w$.
2. O_w is *nested*: whenever $S_1, S_2 \in O_w$, $S_1 \subseteq S_2$ or $S_2 \subseteq S_1$, i.e., O_w is linearly ordered by \subseteq .
3. O_w is closed under non-empty unions.
4. O_w is closed under non-empty intersections.

The intuition behind O_w is that the worlds “around” w are stratified according to how far away they are from w . The innermost sphere is just w by itself, i.e., the set $\{w\}$: w is closer to w than the worlds in any other sphere. If $S \subsetneq S'$, then the worlds in $S' \setminus S$ are further way from w than the worlds in S : $S' \setminus S$ is the “layer” between the S and the worlds outside of S' . In particular, we have to think of the spheres as containing all the worlds within their outer surface; they are not just the individual layers.

The diagram in [Figure 1](#) corresponds to the sphere model with $W = \{w, w_1, \dots, w_7\}$, $V(p) = \{w_5, w_6, w_7\}$. The innermost sphere $S_1 = \{w\}$. The closest worlds to w are w_1, w_2, w_3 , so the next larger sphere is $S_2 = \{w, w_1, w_2, w_3\}$.

The worlds further out are w_4, w_5, w_6 , so the outermost sphere is $S_3 = \{w, w_1, \dots, w_6\}$. The system of spheres around w is $O_w = \{S_1, S_2, S_3\}$. The world w_7 is not in any sphere around w . The closest worlds in which p is true are w_5 and w_6 , and so the smallest p -admitting sphere is S_3 .

To define satisfaction of a formula φ at world w in a sphere model \mathfrak{M} , $\mathfrak{M}, w \Vdash \varphi$, we expand the definition for modal **formulas** to include a clause for $\psi \Box \rightarrow \chi$:

Definition min.2. $\mathfrak{M}, w \Vdash \psi \Box \rightarrow \chi$ iff either

1. For all $u \in \bigcup O_w$, $\mathfrak{M}, u \not\Vdash \chi$, or
2. For some $S \in O_w$,
 - a) $\mathfrak{M}, u \Vdash \psi$ for some $u \in S$, and
 - b) for all $v \in S$, either $\mathfrak{M}, v \not\Vdash \psi$ or $\mathfrak{M}, v \Vdash \chi$.

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sphere-vac
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sphere-nonvac

According to this definition, $\mathfrak{M}, w \Vdash \psi \Box \rightarrow \chi$ iff either the antecedent ψ is false everywhere in the spheres around w , or there is a sphere S where ψ is true, and the material conditional $\psi \rightarrow \chi$ is true at all worlds in that “ ψ -admitting” sphere. Note that we didn’t require in the definition that S is the *innermost* ψ -admitting sphere, contrary to what one might expect from the intuitive explanation. But if the condition in (2) is satisfied for some sphere S , then it is also satisfied for all spheres S contains, and hence in particular for the innermost sphere.

Note also that the definition of sphere models does not require that there *is* an innermost ψ -admitting sphere: we may have an infinite sequence $S_1 \supseteq S_2 \supseteq \dots \supseteq \{w\}$ of ψ -admitting spheres, and hence no innermost ψ -admitting spheres. In that case, $\mathfrak{M}, w \Vdash \psi \Box \rightarrow \chi$ iff $\psi \rightarrow \chi$ holds throughout the spheres S_i, S_{i+1}, \dots , for some i .

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Bibliography