min.1 Contraposition

Material and strict conditionals are equivalent to their contrapositives. Counterfactuals are not. Here is an example due to Kratzer:

If Goethe hadn’t died in 1832, he would (still) be dead now.

If Goethe weren’t dead now, he would have died in 1832.

The first sentence is true: humans don’t live hundreds of years. The second is clearly false: if Goethe weren’t dead now, he would be still alive, and so couldn’t have died in 1832.

Example min.1. The sphere semantics invalidates contraposition, i.e., we have $p \rightarrow q \nRightarrow \neg q \rightarrow \neg p$. Think of $p$ as “Goethe didn’t die in 1832” and $q$ as “Goethe is dead now.” We can capture this in a model $\mathfrak{M}_1 = \langle W, O, V \rangle$ with $W = \{w, w_1, w_2\}$, $O = \{\{w\}, \{w, w_1\}, \{w, w_1, w_2\}\}$, $V(p) = \{w_1, w_2\}$ and $V(q) = \{w, w_1\}$. So $w$ is the actual world where Goethe died in 1832 and is still dead; $w_1$ is the (close) world where Goethe died in, say, 1833, and is still dead; and $w_2$ is a (remote) world where Goethe is still alive. There is a $p$-admitting sphere $S = \{w, w_1\}$ and $p \rightarrow q$ is true at all worlds in it, so $\mathfrak{M}_1, w \models p \rightarrow q$. However, the $\neg q$-admitting sphere $\{w, w_1, w_2\}$ contains a world, namely $w_2$, where $q$ is false and $p$ is true, so $\mathfrak{M}_1, w_2 \nRightarrow \neg q \rightarrow \neg p$. 

Figure 1: Counterexample to contraposition