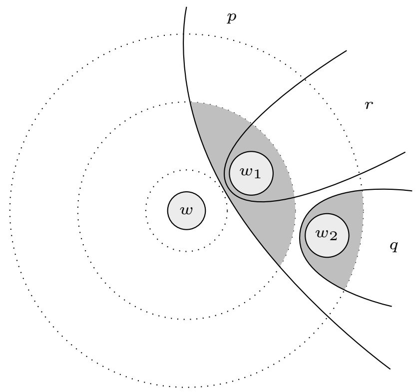

Figure 1: Counterexample to antecedent strengthening

cnt:min:agg:
fig:antecedent-strengthening

min.1 Antecedent Strengthenng

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sec

"Strengthening the antecedent" refers to the inference $\varphi \to \chi \vDash (\varphi \land \psi) \to \chi$. It is valid for the material conditional, but invalid for counterfactuals. Suppose it is true that if I were to strike this match, it would light. (That means, there is nothing wrong with the match or the matchbook surface, I will not break the match, etc.) But it is not true that if I were to light this match in outer space, it would light. So the following inference is invalid:

> I the match were struck, it would light.
>
> Therefore, if the match were struck in outer space, it would light.

The Lewis-Stalnaker account of conditionals explains this: the closest world where I light the match and I do so in outer space is much further removed from the actual world than the closest world where I light the match is. So although it's true that the match lights in the latter, it is not in the former. And that is as it schould be.

Example min.1. The sphere semantics invalidates the inference, i.e., we have $p \mathrel{\Box\!\!\to} r \nvDash (p \land q) \mathrel{\Box\!\!\to} r$. Consider the model $\mathfrak{M} = \langle W, O, V \rangle$ where $W = \{w, w_1, w_2\}$, $O_w = \{\{w\}, \{w, w_1\}, \{w, w_1, w_2\}\}$, $V(p) = \{w_1, w_2\}$, $V(q) = \{w_2\}$, and $V(r) = \{w_1\}$. There is a p-admitting sphere $S = \{w, w_1\}$ and $p \to r$ is true at all worlds in it, so $\mathfrak{M}, w \Vdash p \mathrel{\Box\!\!\to} r$. There is also a $(p \land q)$-admitting sphere $S' = \{w, w_1, w_2\}$ but $\mathfrak{M}, w_2 \nVdash (p \land q) \to r$, so $\mathfrak{M}, w \nVdash (p \land q) \mathrel{\Box\!\!\to} r$ (see Figure 1).

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Bibliography