The Material Conditional

In its simplest form in English, a conditional is a sentence of the form “If . . . then . . .,” where the . . . are themselves sentences, such as “If the butler did it, then the gardener is innocent.” In introductory logic courses, we earn to symbolize conditionals using the $\rightarrow$ connective: symbolize the parts indicated by . . . , e.g., by formulas $\varphi$ and $\psi$, and the entire conditional is symbolized by $\varphi \rightarrow \psi$.

The connective $\rightarrow$ is truth-functional, i.e., the truth value—$T$ or $F$—of $\varphi \rightarrow \psi$ is determined by the truth values of $\varphi$ and $\psi$: $\varphi \rightarrow \psi$ is true iff $\varphi$ is false or $\psi$ is true, and false otherwise. Relative to a truth value assignment $v$, we define $v \models \varphi \rightarrow \psi$ iff $v \not\models \varphi$ or $v \models \psi$. The connective $\rightarrow$ with this semantics is called the material conditional.

This definition results in a number of elementary logical facts. First of all, the deduction theorem holds for the material conditional:

$$\text{If } \Gamma, \varphi \models \psi \text{ then } \Gamma \models \varphi \rightarrow \psi \quad (1)$$

It is truth-functional: $\varphi \rightarrow \psi$ and $\neg \varphi \lor \psi$ are equivalent:

$$\varphi \rightarrow \psi \models \neg \varphi \lor \psi \quad (2)$$

$$\neg \varphi \lor \psi \models \varphi \rightarrow \psi \quad (3)$$

A material conditional is entailed by its consequent and by the negation of its antecedent:

$$\psi \models \varphi \rightarrow \psi \quad (4)$$

$$\neg \varphi \models \varphi \rightarrow \psi \quad (5)$$

A false material conditional is equivalent to the conjunction of its antecedent and the negation of its consequent: if $\varphi \rightarrow \psi$ is false, $\varphi \land \neg \psi$ is true, and vice versa:

$$\neg (\varphi \rightarrow \psi) \models \varphi \land \neg \psi \quad (6)$$

$$\varphi \land \neg \psi \models \neg (\varphi \rightarrow \psi) \quad (7)$$

The material conditional supports modus ponens:

$$\varphi, \varphi \rightarrow \psi \models \psi \quad (8)$$

The material conditional agglomerates:

$$\varphi \rightarrow \psi, \varphi \rightarrow \chi \models \varphi \rightarrow (\psi \land \chi) \quad (9)$$

We can always strengthen the antecedent, i.e., the conditional is monotonic:

$$\varphi \rightarrow \psi \models (\varphi \land \chi) \rightarrow \psi \quad (10)$$
The material conditional is transitive, i.e., the chain rule is valid:

$$\varphi \rightarrow \psi, \psi \rightarrow \chi \models \varphi \rightarrow \chi \quad (11)$$

The material conditional is equivalent to its contrapositive:

$$\varphi \rightarrow \psi \models \neg \psi \rightarrow \neg \varphi \quad (12)$$
$$\neg \psi \rightarrow \neg \varphi \models \varphi \rightarrow \psi \quad (13)$$

These are all useful and unproblematic inferences in mathematical reasoning. However, the philosophical and linguistic literature is replete with purported counterexamples to the equivalent inferences in non-mathematical contexts. These suggest that the material conditional $\rightarrow$ is not—or at least not always—the appropriate connective to use when symbolizing English “if . . . then . . .” statements.

**Photo Credits**

**Bibliography**