In its simplest form in English, a conditional is a sentence of the form “If . . . then . . .,” where the . . . are themselves sentences, such as “If the butler did it, then the gardener is innocent.” In introductory logic courses, we learn to symbolize conditionals using the → connective: symbolize the parts indicated by . . . , e.g., by formulas ϕ and ψ, and the entire conditional is symbolized by ϕ → ψ.

The connective → is truth-functional, i.e., the truth value—T or F—of ϕ→ψ is determined by the truth values of ϕ and ψ: ϕ → ψ is true iff ϕ is false or ψ is true, and false otherwise. Relative to a truth value assignment v, we define v ⊨ ϕ → ψ iff v ⊭ ϕ or v ⊨ ψ. The connective → with this semantics is called the material conditional.

This definition results in a number of elementary logical facts. First of all, the deduction theorem holds for the material conditional:

If Γ, ϕ ⊨ ψ then Γ ⊨ ϕ → ψ (1)

It is truth-functional: ϕ → ψ and ¬ϕ V ψ are equivalent:

ϕ → ψ ⊨ ¬ϕ V ψ (2)
¬ϕ V ψ ⊨ ϕ → ψ (3)

A material conditional is entailed by its consequent and by the negation of its antecedent:

ψ ⊨ ϕ → ψ (4)
¬ϕ ⊨ ϕ → ψ (5)

A false material conditional is equivalent to the conjunction of its antecedent and the negation of its consequent: if ϕ → ψ is false, ϕ ∧ ¬ψ is true, and vice versa:

¬(ϕ → ψ) ⊨ ϕ ∧ ¬ψ (6)
ϕ ∧ ¬ψ ⊨ ¬(ϕ → ψ) (7)

The material conditional supports modus ponens:

ϕ, ϕ → ψ ⊨ ψ (8)

The material conditional agglomerates:

ϕ → ψ, ϕ → χ ⊨ ϕ → (ψ ∧ χ) (9)

We can always strengthen the antecedent, i.e., the conditional is monotonic:

ϕ → ψ ⊨ (ϕ ∧ χ) → ψ (10)
The material conditional is transitive, i.e., the chain rule is valid:

\[ \varphi \to \psi, \psi \to \chi \vdash \varphi \to \chi \quad (11) \]

The material conditional is equivalent to its contrapositive:

\[ \varphi \to \psi \vdash \neg \psi \to \neg \varphi \quad (12) \]

\[ \neg \psi \to \neg \varphi \vdash \varphi \to \psi \quad (13) \]

These are all useful and unproblematic inferences in mathematical reasoning. However, the philosophical and linguistic literature is replete with purported counterexamples to the equivalent inferences in non-mathematical contexts. These suggest that the material conditional \( \to \) is not—or at least not always—the appropriate connective to use when symbolizing English “if . . . then . . .” statements.

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Bibliography