rec.1  Primitive Recursion Notations

One advantage to having the precise inductive description of the primitive recursive functions is that we can be systematic in describing them. For example, we can assign a “notation” to each such function, as follows. Use symbols zero, succ, and $P^m_i$ for zero, successor, and the projections. Now suppose $f$ is defined by composition from a $k$-place function $h$ and $n$-place functions $g_0$, \ldots, $g_{k-1}$, and we have assigned notations $H$, $G_0$, \ldots, $G_{k-1}$ to the latter functions. Then, using a new symbol $\text{Comp}_{k,n}$, we can denote the function $f$ by $\text{Comp}_{k,n}[H, G_0, \ldots, G_{k-1}]$. For the functions defined by primitive recursion, we can use analogous notations of the form $\text{Rec}_k[G, H]$, where $k+1$ is the arity of the function being defined. With this setup, we can denote the addition function by

$$\text{Rec}_2[P^1_0, \text{Comp}_{1,3}\text{[suc}, P^3_2]].$$

Having these notations sometimes proves useful.

**Problem rec.1.** Give the complete primitive recursive notation for mult.

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**Bibliography**