rec.1 Primitive Recursion Notations

cmp:rec:not:sec One advantage to having the precise inductive description of the primitive recursive functions is that we can be systematic in describing them. For example, we can assign a "notation" to each such function, as follows. Use symbols zero, succ, and P^n_i for zero, successor, and the projections. Now suppose f is defined by composition from a k-place function h and n-place functions g_0, …, g_{k-1}, and we have assigned notations H, G_0, …, G_{k-1} to the latter functions. Then, using a new symbol $\mathrm{Comp}_{k,n}$, we can denote the function f by $\mathrm{Comp}_{k,n}[H, G_0, \dots, G_{k-1}]$. For the functions defined by primitive recursion, we can use analogous notations of the form $\mathrm{Rec}_k[G, H]$, where $k+1$ is the arity of the function being defined. With this setup, we can denote the addition function by

$$\mathrm{Rec}_2[P^1_0, \mathrm{Comp}_{1,3}[\mathrm{succ}, P^3_2]].$$

Having these notations sometimes proves useful.

Problem rec.1. Give the complete primitive recursive notation for mult.

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Bibliography