

rec.1 The Normal Form Theorem

cmp:rec:nft:
sec
cmp:rec:nft:
thm:kleene-nf

Theorem rec.1 (Kleene’s Normal Form Theorem). *There is a primitive recursive relation $T(e, x, s)$ and a primitive recursive function $U(s)$, with the following property: if f is any partial recursive function, then for some e ,*

$$f(x) \simeq U(\mu s T(e, x, s))$$

for every x .

The proof of the normal form theorem is involved, but the basic idea is simple. Every partial recursive function has an *index* e , intuitively, a number coding its program or definition. If $f(x) \downarrow$, the computation can be recorded systematically and coded by some number s , and that s codes the computation of f on input x can be checked primitive recursively using only x and the definition e . This means that T is primitive recursive. Given the full record of the computation s , the “upshot” of s is the value of $f(x)$, and it can be obtained from s primitive recursively as well.

The normal form theorem shows that only a single unbounded search is required for the definition of any partial recursive function. We can use the numbers e as “names” of partial recursive functions, and write φ_e for the function f defined by the equation in the theorem. Note that any partial recursive function can have more than one index—in fact, every partial recursive function has infinitely many indices.

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Bibliography