lam.1 Lambda Representable Functions are Computable

cmp:lam:cmp:sec

cmp:lam:cmp:thm:lambda-computable

Theorem lam.1. *If a partial function f is represented by a lambda term, it is computable.*

Proof. Suppose a function f, is represented by a lambda term X. Let us describe an informal procedure to compute f. On input $m_0, \ldots, m_{n-1}$, write down the term $X\overline{m}_0 \ldots \overline{m}_{n-1}$. Build a tree, first writing down all the one-step reductions of the original term; below that, write all the one-step reductions of those (i.e., the two-step reductions of the original term); and keep going. If you ever reach a numeral, return that as the answer; otherwise, the function is undefined.

An appeal to Church's thesis tells us that this function is computable. A better way to prove the theorem would be to give a recursive description of this search procedure. For example, one could define a sequence primitive recursive functions and relations, "IsASubterm," "Substitute," "ReducesToInOneStep," "ReductionSequence," "Numeral," etc. The partial recursive procedure for computing $f(m_0, \ldots, m_{n-1})$ is then to search for a sequence of one-step reductions starting with $X\overline{m_0} \ldots \overline{m_{n-1}}$ and ending with a numeral, and return the number corresponding to that numeral. The details are long and tedious but otherwise routine. □

Photo Credits

Bibliography