

## lam.1 Fixed-Point Combinators

cmp:lam:fix:  
sec Suppose you have a lambda term  $g$ , and you want another term  $k$  with the property that  $k$  is  $\beta$ -equivalent to  $gk$ . Define terms

$$\text{diag}(x) = xx$$

and

$$l(x) = g(\text{diag}(x))$$

using our notational conventions; in other words,  $l$  is the term  $\lambda x. g(xx)$ . Let  $k$  be the term  $ll$ . Then we have

$$\begin{aligned} k &= (\lambda x. g(xx))(\lambda x. g(xx)) \\ &\triangleright g((\lambda x. g(xx))(\lambda x. g(xx))) \\ &= gk. \end{aligned}$$

If one takes

$$Y = \lambda g. ((\lambda x. g(xx))(\lambda x. g(xx)))$$

then  $Yg$  and  $g(Yg)$  reduce to a common term; so  $Yg \equiv_{\beta} g(Yg)$ . This is known as “Curry’s combinator.” If instead one takes

$$Y = (\lambda xg. g(xgx))(\lambda xg. g(xgx))$$

then in fact  $Yg$  reduces to  $g(Yg)$ , which is a stronger statement. This latter version of  $Y$  is known as “Turing’s combinator.”

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## Bibliography