Suppose you have a lambda term $g$, and you want another term $k$ with the property that $k$ is $\beta$-equivalent to $gk$. Define terms

$$\text{diag}(x) = xx$$

and

$$l(x) = g(\text{diag}(x))$$

using our notational conventions; in other words, $l$ is the term $\lambda x. g(xx)$. Let $k$ be the term $ll$. Then we have

$$k = (\lambda x. g(xx))(\lambda x. g(xx))$$


$$g((\lambda x. g(xx))(\lambda x. g(xx)))$$

$$= gk.$$ 

If one takes

$$Y = \lambda g. ((\lambda x. g(xx))(\lambda x. g(xx)))$$

then $Yg$ and $g(Yg)$ reduce to a common term; so $Yg \equiv \beta g(Yg)$. This is known as “Curry’s combinator.” If instead one takes

$$Y = (\lambda xg. g(xxg))(\lambda xg. g(xxg))$$

then in fact $Yg$ reduces to $g(Yg)$, which is a stronger statement. This latter version of $Y$ is known as “Turing’s combinator.”

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**Bibliography**