Lemma lam.1. The lambda representable functions are closed under composition.

Proof. Suppose $f$ is defined by composition from $h$, $g_0$, ..., $g_{k-1}$. Assuming $h$, $g_0$, ..., $g_{k-1}$ are represented by $\overline{h}$, $\overline{g_0}$, ..., $\overline{g_{k-1}}$, respectively, we need to find a term $\overline{f}$ representing $f$. But we can simply define $\overline{f}$ by

$$\overline{f}(x_0, \ldots, x_{i-1}) = \overline{h}(\overline{g_0}(x_0, \ldots, x_{i-1}), \ldots, \overline{g_{k-1}}(x_0, \ldots, x_{i-1})).$$

In other words, the language of the lambda calculus is well suited to represent composition. \qed

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Bibliography