lam.1 Lambda Representable Functions Closed under Composition

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 ${\bf Lemma\ lam.1.}\ The\ lambda\ representable\ functions\ are\ closed\ under\ composition.$

Proof. Suppose f is defined by composition from h, g_0, \ldots, g_{k-1} . Assuming h, g_0, \ldots, g_{k-1} are represented by $\overline{h}, \overline{g_0}, \ldots, \overline{g_{k-1}}$, respectively, we need to find a term \overline{f} representing f. But we can simply define \overline{f} by

$$\overline{f}(x_0,\ldots,x_{l-1}) = \overline{h}(\overline{g_0}(x_0,\ldots,x_{l-1}),\ldots,\overline{g_{k-1}}(x_0,\ldots,x_{l-1})).$$

In other words, the language of the lambda calculus is well suited to represent composition. $\hfill\Box$

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Bibliography