lam.1 Lambda Representable Functions Closed under Composition

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Lemma lam.1. *The lambda representable functions are closed under composition.*

Proof. Suppose f is defined by composition from h, g_0, ..., g_{k-1}. Assuming h, g_0, ..., g_{k-1} are represented by $\overline{h}$, $\overline{g_0}$, ..., $\overline{g_{k-1}}$, respectively, we need to find a term $\overline{f}$ representing f. But we can simply define $\overline{f}$ by

$$\overline{f}(x_0, \ldots, x_{l-1}) = \overline{h}(\overline{g_0}(x_0, \ldots, x_{l-1}), \ldots, \overline{g_{k-1}}(x_0, \ldots, x_{l-1})).$$

In other words, the language of the lambda calculus is well suited to represent composition. □

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Bibliography