**Lemma 1.** The lambda representable functions are closed under composition.

*Proof.* Suppose $f$ is defined by composition from $h, g_0, \ldots, g_{k-1}$. Assuming $h, g_0, \ldots, g_{k-1}$ are represented by $\bar{h}, \bar{g}_0, \ldots, \bar{g}_{k-1}$, respectively, we need to find a term $\bar{f}$ representing $f$. But we can simply define $\bar{f}$ by

$$\bar{f}(x_0, \ldots, x_{l-1}) = \bar{h}(\bar{g}_0(x_0, \ldots, x_{l-1}), \ldots, \bar{g}_{k-1}(x_0, \ldots, x_{l-1})).$$

In other words, the language of the lambda calculus is well suited to represent composition. □

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**Bibliography**