

## lam.1 The Church-Rosser Property

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thm:church-rosser **Theorem lam.1.** *Let  $M$ ,  $N_1$ , and  $N_2$  be terms, such that  $M \triangleright N_1$  and  $M \triangleright N_2$ . Then there is a term  $P$  such that  $N_1 \triangleright P$  and  $N_2 \triangleright P$ .*

**Corollary lam.2.** *Suppose  $M$  can be reduced to normal form. Then this normal form is unique.*

*Proof.* If  $M \triangleright N_1$  and  $M \triangleright N_2$ , by the previous theorem there is a term  $P$  such that  $N_1$  and  $N_2$  both reduce to  $P$ . If  $N_1$  and  $N_2$  are both in normal form, this can only happen if  $N_1 = P = N_2$ .  $\square$

Finally, we will say that two terms  $M$  and  $N$  are  $\beta$ -equivalent, or just *equivalent*, if they reduce to a common term; in other words, if there is some  $P$  such that  $M \triangleright P$  and  $N \triangleright P$ . This is written  $M \equiv N$ . Using [Theorem lam.1](#), you can check that  $\equiv$  is an equivalence relation, with the additional property that for every  $M$  and  $N$ , if  $M \triangleright N$  or  $N \triangleright M$ , then  $M \equiv N$ . (In fact, one can show that  $\equiv$  is the *smallest* equivalence relation having this property.)

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## Bibliography