lam.1 The Church-Rosser Property

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Theorem lam.1. *Let M, N_1, and N_2 be terms, such that $M \triangleright N_1$ and $M \triangleright N_2$. Then there is a term P such that $N_1 \triangleright P$ and $N_2 \triangleright P$.*

Corollary lam.2. *Suppose M can be reduced to normal form. Then this normal form is unique.*

Proof. If $M \triangleright N_1$ and $M \triangleright N_2$, by the previous theorem there is a term P such that N_1 and N_2 both reduce to P. If N_1 and N_2 are both in normal form, this can only happen if $N_1 = P = N_2$. □

Finally, we will say that two terms M and N are *β-equivalent*, or just *equivalent*, if they reduce to a common term; in other words, if there is some P such that $M \triangleright P$ and $N \triangleright P$. This is written $M \equiv N$. Using Theorem lam.1, you can check that $\equiv$ is an equivalence relation, with the additional property that for every M and N, if $M \triangleright N$ or $N \triangleright M$, then $M \equiv N$. (In fact, one can show that $\equiv$ is the *smallest* equivalence relation having this property.)

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Bibliography